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Colburn's First Part.

THE

FIRST BOOK

OF

ARITHMETIC.

BY

DANA P. COLBURN,

PRINCIPAL OF THE RHODE ISLAND STATE NORMAL SCHOOL, AND
AUTHOR OF "ARITHMETIC AND ITS APPLICATIONS."

PHILADELPHIA:
H. COWPERTHWAIT & CO.
1856.

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PREFACE.

The Decimal System of Numbers is one of the most perfect things of man's invention. It is so simple, that a child can understand it, yet so comprehensive that it includes all possible numbers, represents them all by ten simple characters and a point, and bases all numerical operations on the combinations of the first ten, or primitive numbers.

These primitive combinations can easily be determined. 1 can be added to each number from 1 to 10; so can each of the first ten numbers, and on these will depend all possible combinations in addition. For since 3 + 2 = 5, we have 30 + 20 = 50, 300 + 200 = 500, &c., 23 + 2 = 25, 193 + 2 = 195, &c.

But subtraction is so closely connected with addition, that, as far as the primitive numbers are concerned, a knowledge of one implies a knowledge of the other. What, for instance, are 2 + 3 = 5, 2 from 5 = 3, 3 from 5 = 2, 5 = 3 more than 2, &c., but different forms of expressing the idea that 5 is made up of 2 and 3?

It is certainly more philosophical to present these various forms in such connexion, so as to show their mutual dependence, and thus secure thoroughness from the outset, than to present one form through a long series of lessons, and then another form through another long series of lessons, as though they have no connexion with each other. The invariable experience of teachers who have given both methods a fair trial is, that elementary addition and subtraction can thus be taught together, with greater ease than either can be taught by itself.

All that has been said of addition and subtraction applies with equal force to multiplication and division. Hence, the varied combinations of the primitive numbers ought to be mastered by the pupil before he attempts those of the derived numbers; and when the latter are introduced, it should be in such a way as to show their dependence on the former.

Another reason for this careful elementary instruction is found in the fact that a child will readily understand and solve a problem involving small numbers, when a similar one, involving large numbers, will be entirely beyond his comprehension. There can be no doubt, then, that his attention should be confined to small numbers, till his mathematical powers are so far developed as to enable him to use large numbers understandingly.

The lessons of a First Book of Arithmetic should be based on such principles.

They should be so arranged as to illustrate in an easy and familiar manner the nature and uses of numbers and of numerical operations, to call into exercise and discipline the mental powers, form accurate habits of thought and investigation, impart a just self-reliance, develop a power of following closely the most rigid reasoning processes, and lay a sure foundation for future progress in mathematical studies.

They ought to be simple in their beginnings, gradual in their developments, interesting in their problems, varied in their exercises, and so connected, that each shall follow naturally from those that go before, and prepare the way for those that come after.

Moreover, they should embody such a variety and extent of exercises as to include all the essential principles of Arithmetic, and thus prepare the way for any advanced treatise, and even give those who have no further opportunity for study *in school*, such discipline as will enable them to meet the demands of real life.

The author has endeavoured to prepare this treatise in accordance with these views. In its preparation he has drawn freely from the "First Steps in Numbers," and the "Decimal System of Numbers," both issued some years since, — the the latter of which was written by himself, and the former conjointly with Mr. GEORGE A. WALTON, now of Lawrence, Mass.

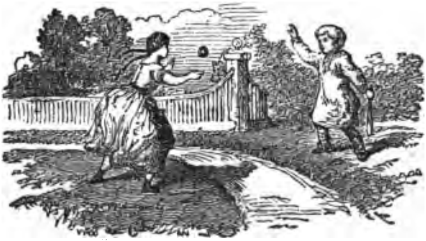
Whatever may be its merits or defects, it is the result of much careful thought and study, of considerable experience as a Teacher, and of an honest effort to arrange such a course of lessons as shall aid in developing the youthful mind, and in forming correct habits of study.

DANA P. COLBURN.

PROVIDENCE, *July*, 1856.

COLBURN'S FIRST PART.

LESSON I.



One boy. 1 boy. *1 boy.*

One girl. 1 girl. *1 girl.*

One ball. 1 ball. *1 ball.*

One. 1. *1*. I.

1. How many pictures are there on this page?
2. How many boys do you see in the picture?
3. How many girls do you see in the picture?
4. How many balls do you see in the picture?
5. How many thumbs have you on your right hand?
6. How many thumbs have you on your left hand?

Note to the Teacher.—The pictures are not designed to take the place of exercises with visible objects, but rather as additional illustrations of the numbers introduced. With young classes, the Teacher should give such easy lessons as the following, making use of the most familiar objects as counters.

Even though the pupils have used numbers somewhat, such lessons will make them better acquainted with their nature, and will thus ensure a more rapid advancement.

Oral Lesson.—Teacher, taking a book, asks: "What have I in my hand?" *Ans.*—"A book." "How many books?" *Ans.*—"One book." "How many pencils do I show you?" *Ans.*—"One pencil." "How many chairs do I point at?" *Ans.*—"One chair." "How many desks?" *Ans.*—"One desk."

Tell the class to point to one boy, to one girl, to one window, &c., &c.

The mark 1 (making it on the board) means one, as, 1 dog, 1 book. When writing, we make it thus:—*1 dog, 1 book.*

Note.—An oral lesson like the following may precede Lesson II.

Oral Lesson.—"How many pencils have I in my right hand?" *Ans.*—"One pencil." "How many in my left hand?" *Ans.*—"One pencil." "How many in both?" *Ans.*—"Two pencils." "How many pieces of chalk have I in my right hand?" *Ans.*—"One." "In my left?" *Ans.*—"One." "In both?" *Ans.*—"Two." "How many fingers do I hold up?" *Ans.*—"Two."

"One pen and one pen are, how many pens?" "If I lay down one pen, how many shall I have left?" Lay down one pen, and show the remaining one. "How many more must I get to have two?" Taking two pens in the right hand and one in the left, ask: "How many pens have I in my right hand?" "How many in my left hand?" "How many more in my right hand than in my left?" "How many less in my left hand than in my right hand?" "If I should pass one from my right hand to my left hand, how many would there be in my right hand?" "How many would there be in my left?"

Ask such questions as these, illustrating each by familiar objects, and continuing the exercise, till the numbers two, and one, and their relations to each other, are perfectly understood.

LESSON II.

Two boys. 2 boys. *2 boys*

Two soap-bubbles. 2 soap-bubbles. *2 soap-bubbles.*

Two. 2. *2*. II.

The mark 2, or *2*, is called the *figure two.*

How many boys do you see in the picture?
How many soap-bubbles do you see in the picture?

A. To the Teacher.—The following questions should first be asked by substituting concrete in place of the abstract numbers. Thus: "How many apples are 1 apple and 1 apple? 1 pear and how many pears are two pears?" &c. The pupils should be taught to make such changes for themselves. The work is not, however, mastered till the abstract numbers and operations are mastered.

1. How many are 1 and 1?
2. 1 and how many are 2?
3. 1 from 2 leaves how many?
4. How many must be taken from 2 to leave 1?
5. How many more are 2 than 1?

4. A STORY ABOUT CARRIE.—Carrie was a bright-eyed little girl who lived in a village. One day she cut out 2 paper dolls, and the next day she cut out 1 more. She then gave 1 to her playmate, Martha, who came to see her, and 1 to Maria. She afterwards cut out 2 more, but through carelessness, let 1 fall into the fire, when her mother cut out 1 very nice one, and gave it to her. How many had she then?

TO THE TEACHER.—Make additional problems, and encourage the pupils to do it for themselves, arranging them somewhat in the form of stories, to increase their interest. One problem proposed by a pupil, and solved by a class, will be of more value in an educational view than many proposed by a teacher or author.

LESSON IV.

Four reapers. 4 reapers. *4 reapers.*

Four. 4. *4*. IV.

The mark 4, or *4*, is called the *figure four.*

NOTE.—It should be made a part of the lesson for the pupil to write out the exercises in abstract numbers. He will thus learn to use figures and mathematical signs, and to write out arithmetical work neatly and correctly.

EXPLANATION.—A cross made thus, +, is sometimes used in place of "and" in such questions as, how many are 1 and 2? In like manner, "2 + 2 are 4" mean the same as "2 and 2 are 4." This sign is also called *plus*, and sometimes the *sign of addition*.

A. 1. 3+1? 3. 2+2? 5. 1+1+2?
2. 1+3? 4. 1+2+1? 6. 2+1+1?

EXPLANATION. — Read and perform the following questions, and similar ones throughout the book, as though the words "how many" were put in the place of the star. Thus, the question "2 + * = 4?" means the same as "2 and how many are 4?"

B. 1. 2+* are 4? 4. 1+1+* are 4?
2. 1+* are 4? 5. 1+2+* are 4?
3. 3+* are 4? 6. 2+1+* are 4?

C. 1. 1 from 4? 3. 3 from 4?
2. 2 from 4? 4. 1 from 3?

D. How many more are—
1. 4 than 3? 3. 4 than 2?
2. 4 than 1? 4. 3 than 1?

E. 1. George has 2 apples, and Rufus has 1. How many have both? How many more has George than Rufus?

2. Edward had 2 marbles, and his father gave him a cent, with which he bought 2 more. How many had he then?

3. He afterwards lost 1, and gave away 2. How many had he left?

4. Jane had 1 picture-book, and on her birth-day her father gave her 1 more; her mother gave her 1, and her uncle Henry sent her 1, which was very pretty. How many had she then?

5. There were 3 robins on a cherry-tree, but 1 of them flew away, and 2 others came to the tree. A naughty boy threw a stone to knock down some cherries, which so frightened the robins that 4 of them flew away. How many robins were left on the tree?

LESSON V.

Five toy-horses. 5 toy-horses. *5 toy-horses.*

Five. 5. *5*. V.

The mark 5, or *5*, is called the *figure five.*

A. 1. 4+1?
2. 1+4?
3. 3+2?
4. 2+3?
5. 3+1+1?
6. 1+2+2?
7. 2+1+2?
8. 1+1+3?
9. 2+2+1?

B. **Explanation.** — Two parallel lines drawn thus, =, form what is called the *sign of equality*, which is often used in place of "are" in such cases as "2 + 2 are 4," which would then be written "2 + 2 = 4." This may be read "2 and 2 are 4," or "2 plus 2 are 4," or "2 plus 2 equal 4."

1. 2+*=5?
2. 1+*=5?
3. 2+*=5?
4. 2+1+*=5?
5. 1+1+*=5?
6. 2+2+*=5?

C. 1. 2 from 5?
2. 4 from 5?
3. 1 from 5?
4. 3 from 5?

D. How many more are —

1. 5 than 2?
2. 5 than 4?
3. 5 than 3?
4. 5 than 1?

E. 1. Edwin had 2 cents, but he afterwards found 3, and spent 4. How many had he left?

2. Arthur had a half-dime, which, as you know, is worth just 5 cents. He went to a store and bought some nuts for 2 cents, and some candy for 1 cent, giving in payment his half-dime. How many cents ought he to receive back?

3. Mr. French had 3 black horses, 2 white horses, and 1 grey horse. He sold his grey horse, and 1 of his black ones. How many had he left?

4. Near a village lived a poor woman named Lucy, but everybody called her Aunt Lucy. In the summer she would pick blackberries to sell. One day she picked 3 quarts in one pasture, and 1 in another, when, meeting a gentleman from the village, she sold him 2 quarts. She picked 3 quarts more, and started to go home. On her way, she sold 2 quarts to one man, and 1 quart to another. How many had she left?

LESSON VI.

Six birds. 6 birds. *6 birds*

Six. 6. *6*. VI.

The mark 6, or *6*, **is called the** ***figure six.***

A. 1. 5+1?
2. 1+5?
3. 4+2?
4. 2+4?
5. 3+3?
6. 2+2+2?
7. 1+3+2?
8. 1+2+2?

B. 1. 2+*=6?
2. 3+*=6?
3. 4+*=6?
4. 1+1+1+*=6?
5. 1+2+*=6?
6. 1+1+2+*6?

C. 1. 4 from 6?
2. 1 from 6?
3. 2 from 6?
4. 3 from 6?
5. 5 from 6?
6. 6 from 6?

D. Explanation.—6 less 2 means 6 diminished by 2, or made smaller by 2, which is just the same as "2 from 6." Hence 6 less 2 = 4; 5 less 3 = 2, &c.

1. 6 less 2?
2. 5 less 3?
3. 6 less 3?
4. 5 less 1?
5. 4 less 2?
6. 6 less 4?

E. 1. There were 6 boys at play; 2 of them were flying their kites, and the rest were rolling their hoops. How many were rolling their hoops?

2. A pedler had 6 plaster birds on a tray. 2 of them were painted yellow, 1 of them was painted red and brown, and the rest were painted red and black. How many were painted red and black?

3. A hunter shot 1 partridge, 3 quails, and 2 pigeons. How many birds did he shoot in all?

4. Julia picked 3 white roses, and 3 red ones. How many did she pick in all? She gave 2 red roses and 1 white rose to her teacher, and 1 white rose to her friend Lydia. How many had she?

LESSON VII.

Seven hens. 7 hens. *7 hens.*

Seven. 7. *7.* VII.

The mark 7, or *7*, is called the *figure seven.*

A. 1. 6+1?
2. 1+6?
3. 5+2?
4. 2+5?
5. 3+4?
6. 4+3?
7. 3+2+2?
8. 1+1+5?
9. 2+1+4?

B. 1. 4+*=7?
2. 2+*=7?
3. 2+3+*=7?
4. 2+1+*=7?
5. 1+2+1+*=7?
6. 2+2+2+*=7?

C. 1. 6 from 7?
2. 4 from 7?
3. 2 from 7?
4. 3 from 7?
5. 1 from 7?
6. 5 from 7?

D. 1. 7 less 3?
2. 7 less 1?
3. 7 less 2?
4. 7 less 6?
5. 7 less 5?
6. 7 less 4?

E. 1. A farmer had seven grey hens. He sold 2 of them, and a fox killed 1 of them. How many did he have left? If he should afterwards sell 2 more, and buy 4 small white hens, how many would he then have?

2. Alfred had a half-dime and 4 cents; but he exchanged the half-dime for its value in cents. How many cents did he then have? He was so unfortunate as to lose 3 of his cents, and he gave 3 more for a three-cent piece. How many cents had he then left?

3. 6 boys were at play together; 1 of them got hurt, and went home, and 2 were called away by their friends; but very soon 4 more boys came out to play. These played together till all but 3 got tired, and sat down to rest. How many sat down to rest?

LESSON VIII.

Eight persons. 8 persons. *8 persons.*

Eight. 8. *8.* VIII.

The mark 8, or *8*, is called the *figure eight.*

A. 1. 7+1?
2. 1+7?
3. 6+2?
4. 2+6?
5. 5+3?
6. 3+5?
7. 4+4?
8. 2+2+2+2?

B. 1. 2+*=8?
2. 4+*=8?
3. 3+*=8?
4. 3+2+*=8?
5. 1+1+2+1+*=8?
6. 1+2+3+*=8?

C. 1. 7 from 8?
2. 2 from 8?
3. 4 from 8?
4. 6 from 8?
5. 3 from 8?
6. 5 from 8?

D. How many more are —

1. 8 than 5?
2. 8 than 1?
3. 8 than 6?
4. 8 than 3?
5. 8 than 2?
6. 8 than 4?

E. Explanation. — A single mark made like a dash, thus, —, is often used in place of the word "less." For instance: "8 — 3 = 5" means the same as "8 less 3 = 5."

1. 8—3?
2. 8—7?
3. 8—5?
4. 8—2?
5. 8—6?
6. 8—4?

F. 1. In a ferry-boat were 4 ferrymen, 2 ladies, and 2 gentlemen. How many persons were in the boat?

2. A farmer had 8 little pigs. He sold 2 to one man, and 2 to another. How many had he left?

3. Sarah's mother gave her 3 dresses for her doll, her sister Susan gave her 2, and her aunt Mary gave her enough to make up 8. How many did her aunt Mary give her?

4. Alfred found 3 chestnuts under one tree, 4 under another, and 1 under another. He soon after ate 2, when, having the ill luck to fall, he lost 3. He afterwards found 4 more, when, seeing a pretty squirrel run into a hole in a tree, he put in 3 chestnuts for the squirrel to eat. How many chestnuts had he left?

LESSON IX.

Nine ducklings. 9 ducklings. *9 ducklings.*

Nine. 9. *9.* IX.

The mark 9, or *9*, is called the *figure nine.*

A. 1. 8+1?
2. 1+8?
3. 7+2?
4. 2+7?
5. 6+3?
6. 3+6?
7. 5+4?
8. 4+5?
9. 3+1+3?
10. 2+2+5?
11. 3+2+3?
12. 1+2+4?

B. 1. 5+*=9?
2. 2+*=9?
3. 4+*=9?
4. 3+*=9?
5. 2+5+*=9?
6. 3+5+*=9?
7. 3+3+*=9?
8. 2+2+2+*=9?

C. 1. 8 from 9?
2. 6 from 9?
3. 3 from 9?
4. 7 from 9?
5. 4 from 9?
6. 2 from 9?

D. 1. 9—5?
2. 9—7?
3. 9—6?
4. 9—4—2?
5. 9—2—3?
6. 9—5—2?

E. 1. If a boy should have 6 cents, and receive a present of 2 more, how many would he have? If he should spend 3, and then have 4 given him, how many would he have?

2. Daniel had 3 baskets. The first was a red one, which held 3 quarts; the second was a blue one, which held 4 quarts; the third was a yellow one, which held 2 quarts. How many quarts would all hold?

3. One day he picked so many berries, that he filled them all; but he sold what there was in the blue basket. How many quarts had he left?

4. He emptied the contents of his red basket into his blue basket. How many more quarts would it take to fill it?

5. David had 3 marbles, and Austin had 4; but David found 4, and Austin lost 2. They then agreed to put what they had into a little box. How many marbles did they put into the box?

LESSON X.

Ten herrings. 10 herrings. *10 herrings.*

Ten. 10. *10.* X.

The mark 0 is called the *figure naught*, or *zero*.

A. 1. 9+1?
2. 1+9?
3. 8+2?
4. 2+8?
5. 7+3?
6. 3+7?
7. 6+4?
8. 4+6?
9. 5+5?
10. 4+3+2?
11. 2+3+5?
12. 1+2+2+2+2?
13. 2+2+2+2+2?

B. 1. 2+*=10?
2. 3+*=10?
3. 5+*=10?
4. 4+*=10?
5. 1+4+2+*=10?
6. 3+1+2+*=10?
7. 2+4+3+*=10?
8. 4+1+2+*=10?

C. How many more are—

1. 10 than 6 ?
2. 8 than 3 ?
3. 10 than 5 ?
4. 7 than 4 ?
5. 10 than 7 ?
6. 10 than 8 ?

D. 1. 10—8 ?
2. 10—7 ?
3. 10—6 ?
6. 10—3 ?
5. 10—5+3 ?
6. 10—4—3+6 ?
7. 10—3+2—4 ?
8. 10—4—4+5 ?

E. 1. A hunter shot 3 birds from one flock, 2 from another, and 5 from another. How many did he shoot in all ?

2. Anna says she has 10 picture-books, of which her mother gave her 3, her teacher gave her 1, her aunt gave her 1, her uncle gave her 1, and her father gave her the rest. How many did her father give her ?

3. Edward had a half-dime, a three-cent piece, and 2 cents. How many cents were they worth ? He bought an apple for 2 cents, an orange for 3 cents, some candy for 1 cent, and some raisins for 3 cents. How many cents had he left ?

4. Albert and Timothy went a-fishing one day. Albert caught 3 perch, 2 pickerel, and 4 trout. Timothy caught 2 perch, 3 pickerel, 3 trout, and 2 eels. How many more fish did Timothy catch than Albert ? Albert gave his 4 trout in exchange for Timothy's 2 perch and 3 pickerel. How many fish had each boy then ?

LESSON XI.

Eleven arrows. 11 arrows. *11 arrows.*

Eleven. 11. *11.* XI.

A. 1. 10+1?
2. 9+2?
3. 2+9?
4. 8+3?
5. 3+8?
6. 7+4?
7. 4+7?
8. 6+5?
9. 5+6?
10. 4+2+3?
11. 2+5+4?
12. 2+2+1+2+2+2?
13. 1+2+2+2+1+2?
14. 3+1+3+2+2?

B. 1. 5+*=11?
2. 2+*=11?
3. 4+*=11?
4. 2+2+5+*=11?
5. 3+4+*=11?
6. 3+3+3+*=11?

C. 1. * from 11=5?
2. * from 11=6?
3. * from 11=7?
4. * from 11=4?
5. * from 11=8?
6. * from 11=3?

D. 1. 11—4?
2. 11—2?
3. 11—6?
4. 11—8?
5. 11—5?
6. 11—9?
7. 11—7?
8. 11—3?
9. 11—10?
10. 5+4+2—6?
11. 1+2+4+4—3?
12. 2+5+3—8?
13. 3+1+5—3?
14. 7+4—3—3?

E. 1. A person was shooting arrows at a target, and I observed that when he had shot 3 arrows, and placed another in his bow, there were 7 lying on the ground. How many were there in all?

2. William owned 3 arrows, George owned 2, and Rufus 5. How many did they all own? One afternoon, as they were playing with their bows and arrows, William lost 1 arrow, Rufus lost 1 and broke 1, and George found a very nice one, which some boy had lost. How many arrows had the boys then?

3. One beautiful afternoon in June, Emma and Hannah went out to gather wild flowers, and make boquets. Emma made 4, and Hannah made 5, when they put the rest of their flowers together, and made 2 very pretty boquets. They put them all in a basket, and went home. They gave 3 to Hannah's mother, and 1 to her sister; and they gave 3 to Emma's aunt, and 2 to her teacher; after which, Emma took 1, and Hannah took the rest. How many did Hannah take?

LESSON XII.

Twelve eggs. 12 eggs. 12 eggs.

Twelve. 12. 12. XII.

A. 1. 10+2?
2. 2+10?
3. 9+3?
4. 3+9?
5. 8+4?
6. 4+8?
7. 7+5?
8. 5+7?
9. 6+6?
10. 3+3+3+3?
11. 3+2+3+2?
12. 1+4+3+3?
13. 2+5+2+3?
14. 4+2+2+4?

B. 1. 4+*=12?
2. 5+*=12?
3. 3+*=12?
4. 3+4+3+*=12?
5. 2+1+4+*+12?
6. 2+1+2+*=12?

C. 1. 6 from 12?
2. 9 from 12?
3. 2 from 12?
4. 8 from 12?
5. 10 from 12?
6. 7 from 12?

D. 1. 12—4 ?
2. 12—3 ?
3. 12—7 ?
4. 12—8 ?
5. 12—10 ?
6. 12—9 ?
7. 12—3—4 ?
8. 2+7+3—6 ?
9. 12—5—2 ?
10. 1+8+2—4 ?

E. 1. Alfred found a hen's nest, with a large number of eggs in it. He took out 3, and then took out 4 more, when he found that there were 5 left in the nest. How many were there in the nest at first? When he was putting the eggs back, he carelessly broke 2. How many were then left ?

2. Alice had 11 little chickens, but 2 of them died, 2 of them got lost, and a rat killed 3. How many then remained ?

3. Benjamin earned a half-dime, and found a three-cent piece and 4 cents. He bought a top for 8 cents, after which he received a present of 8 cents. How much money had he then ?

4. Annie had 7 pictures, and Emma had 5. Annie gave away 3 pictures, and Emma received a present of 2; after which Emma lost 3, and Annie found 1. How many pictures had each girl then ? How many had both ?

LESSON XIII.

Thirteen sheep. 13 sheep. *13 sheep.*

Thirteen. 13. *13.* XIII.

A. 1. 10+3?
2. 3+10?
3. 9+4?
4. 4+9?
5. 8+5?
6. 5+8?
7. 7+6?
8. 6+7?
9. 2+3+4+3?
10. 1+4+2+6?
11. 4+3+3+4
12. 1+5+3+4?

B. 1. 3+*=13?
2. 5+*=13?
3. 6+*=13?
4. 3+4+*=13?
5. 4+2+4+*=13?
6. 2+3+4+*=13?

C. 1. 9 from 13?
2. 7 from 13?
3. 4 from 13?
4. 6 from 13?
5. 5 from 13?
6. 8 from 13?

D. 1. 13—*=7?   4. 13—*=9?
2. 13—*=4?   5. 13—*=6?
3. 13—*=8?   6. 13—*=5?

E. 1. Mr. Green owns 13 sheep, and Mr. Al owns 7. How many more does Mr. Green own t Mr. Allen? If Mr. Green should sell 4 sheep Mr. Allen, how many would each have?

2. A farmer had 3 sheep in one pasture, 5 another, and 4 in another; but at night he dr them all into one pen. How many were there the pen? The next day he drove 6 of them i one pasture, 2 into another, and the rest into other. How many did he drive into the last pastu

3. A little boy had 13 marbles. He lost 4, gave away 3, when, finding it was school-time, put the rest into a box. When he came f school, he found his little brother Erastus had b playing with the box, and had lost 3 of the bles. How many were left in the box? His fat afterwards gave him 2 cents, with which he chased 5 marbles. How many marbles had then?

LESSON XIV.

Fourteen barrels. 14 barrels. *14 barrels.*

Fourteen. 14. *14.* XIV.

A. 1. 10+4?
2. 4+10?
3. 9+5?
4. 5+9?
5. 8+6?
6. 6+8?
7. 7+7?
8. 2+1+7+4?
9. 4+2+8?
10. 3+4+2+4?

B. 1. 7+*=14?
2. 5+*=14?
3. 6+*=14?
4. 3+4+*=14?
5. 4+6+*=14?
6. 2+4+3+*=14?

C. 1. 9 from 14?
2. 6 from 14?
3. 4 from 14?
4. 8 from 14?
5. 7 from 14?
6. 4 from 14?

D. 1. 14—4—3?
2. 14—6+3?
3. 14—7+5?
4. 4+5+5—6+3?
5. 8+2+3—5—3?
6. 3+2+4+5—6—3?

E. 1. A teamster had a load of 14 barrels. H unloaded 6 at the store of Shaw & Co., 3 at a railroa depôt, and the rest at the store of Saunders & Brown. How many did he unload at the last place Not long after, he had a load of 13 barrels, and h unloaded 4 of them at one place, and 3 at another after which he took on his truck 8 barrels more How many had he then on his load?

2. Susan has 6 books without pictures, and 7 book with pictures. How many books has she?

3. Austin has 14 books. Waldo had 6. His mother gave him 3, and his father gave him enough t make as many as Austin. How many did his fathe give him?

4. One day, Henry went out to look for chestnuts. He found 6 under one tree, 3 under another and 5 under another. After eating 8 of them, and finding 6 more, he went home, carrying his chestnuts with him. He gave 3 of them to his father, 3 t his mother, 4 to his little sister Lucy, and the rest t his brother Francis. How many did he give t Francis?

LESSON XV.

Fifteen apples. 15 apples. *15 apples.*

Fifteen. 15. *15*. XV.

A. 1. 10+5? 3. 9+6? 5. 8+7?
2. 5+10? 4. 6+9? 6. 7+8?

B. 1. 8 from 15? 4. 7 from 15?
2. 6 from 15? 5. 5 from 15?
3. 10 from 15? 6. 9 from 15?

C. 1. 15−7+3? 4. 8+2+5−7?
2. 15−6+4? 5. 4+3+8−6?
3. 15−9−5? 6. 2+7+5−8+3?

D. 1. 15−*=7? 4. 15−*=5?
2. 15−*=6? 5. 15−*=8?
3. 15−*=9? 6. 14−*=10?

E. 1. Mary found 3 apples under one tree, 4 a under another, and 8 under another. How m did she find in all? As she was bringing the the house, she stopped to play with her kitten, accidentally dropped most of them, as you see i picture. On picking them up, she found that them were bruised a little, and 1 of them, whic kitten played with, was bruised very badly. The were not bruised at all. How many were not br at all?

2. Julia made 4 squares of blue patch-wor squares of brown, and 8 squares of red. How m did she make in all?

3. Hattie hemmed 15 handkerchiefs, 5 of were for her sister Lydia, 2 were for her bro Cyrus, 3 for her father, and the rest for her mo How many did she hem for her mother?

4. Augusta received 4 merit-marks on Mond on Tuesday, 1 on Wednesday, 2 on Thursday, a on Friday, and on Saturday school was not in sion. How many merit-marks did she receive thr the week? How many more than Emeline, obtained but 9 during the week?

LESSON XVI.

Sixteen tents. 16 tents. 16 tents.

Sixteen. 16. 16. XVI.

A. 1. 10+6 ?
2. 6+10 ?
3. 9+7 ?
4. 7+9 ?
5. 8+8 ?

B. 1. 8+*=16 ?
2. 6+*=16 ?
3. 2+5+*=16 ?
4. 3+4+2+*=16 ?
5. 4+2+4+*=16 ?
6. 1+3+2+*=16 ?

C. 1. 16–6 ?
2. 16–9 ?
3. 16–7 ?
4. 16—*=7 ?
5. 16—*=9 ?
6. 16—*=8 ?

D. 1. 14=4+*?
2. 16=6+*?
3. 13=3+*?
4. 12=2+*?
5. 11=1+*?
6. 16=10+*?
7. 13=10+*?
8. 14=10+*?
9. 12=10+*?
10. 11=10+*?

E. 1. 4+2?
2. 14+2?
3. 4+12?
4. 3+3?
5. 13+3?
6. 3+13?
7. 1+4?
8. 11+4?
9. 1+14?

F. 1. On a certain muster-field, there were tents in one row, and 8 in another. How many we there in both rows?

2. Albert was asked how many chestnuts he ha to which he replied, "If I should give my father my mother 4, my little sister Anna 4, and my br ther George 3, I should have but 2 left." How ma chestnuts had he?

3. Lucy read 6 pages of history in the mornin and 10 in the afternoon, but when questioned abo it, she found that she had forgotten all but 4 page How many had she forgotten?

4. I had 3 dollars, and received 6 dollars of o man, 3 of another, and 2 of another, when I pa away 8 dollars, after which I received 4 dolla How many had I then?

LESSON XVII.

Seventeen birds. 17 birds. 17 birds.

Seventeen. 17. 17. XVII.

A. 1. 10+7?
2. 7+10?
3. 8+9?
4. 9+8?
5. 3+5+9?
6. 7+2+8?

B. 1. 4+5+*=17?
2. 3+7+*=17?
3. 4+3+*=17?
4. 4+4+*=17?

C. 1. * from 17=8?
2. * from 17=10?
3. * from 17=7?
4. * from 17=9?

D. 1. 3+1+3?
2. 13+1+3?
3. 3+11+3?
4. 2+2+2?
5. 12+2+2?
6. 2+12+2?

E. 1. 4+*=7 ?
2. 4+*=17 ?
3. 14+*=17 ?
4. 3+*=6 ?
5. 3+*=16 ?
6. 13+*=16 ?
7. 3 from 7 ?
8. 3 from 17 ?
9. 13 from 17 ?
10. 1 from 6 ?
11. 1 from 16 ?
12. 11 from 16 ?

F. 1. Jane gave 6 cents to a beggar-woman, gave 3, Sarah gave 5, and Abby gave 3. How did all give her?

2. The beggar-woman spent 10 cents for b after which Julia gave her 4 cents, Nancy gav 3 cents, and Susan gave her 2 cents. How cents had she then?

3. A gardener picked 8 roses from one b from another, and 2 from another. How man he pick in all? He put 4 of the roses in one b 5 in another, and 3 in another, and the re another. How many did he put in the last bo

4. Samuel bought a quart of molasses f cents, and then had 6 cents left. How many had he at first?

5. He made his molasses into candy, 9 stic which he sold for 7 cents, and the remaining 8 he sold for 6 cents. How many sticks did he How many cents did he receive for it? How more cents did he receive for his candy than h for his molasses?

LESSON XVIII.

Eighteen books. 18 books. *18 books.*

Eighteen. 18. *18.* XVIII.

A. 1. 10+8? 3. 9+9?
2. 8+9?

B. 1. 8+*=18? 4. 2+2+*=14?
2. 9+*=18? 5. 12+2+*=14?
3. 10+*=18? 6. 2+12+*=14?

C. 1. * from 7=3? 4. * from 8=4?
2. * from 17=3? 5. * from 18=4?
3. * from 17=13? 6. * from 18=14?

D. 1. $8-2-2-2$?
2. $18-2-2-2$?
3. $18-12-2-2$?
4. $17-3-3+4$?
5. $9+6+3-4$?
6. $9+3+4-2$?
7. $10+3+4-3$?
8. $18-4-4+3+2$?

E. 1. One "Fourth of July," Robert's father gave him a dime, his mother gave him a half-dime, and his uncle gave him a three-cent piece; but he exchanged them all for their value in cents. How many cents did he receive for them?

2. He paid 8 cents for a bunch of crackers, and 4 cents for torpedoes, and the rest of his money to see some animals, which were exhibited in a tent. How many cents did he pay to see the animals?

3. Mr. Gay owns a garden, a pasture, a wood-lot, and an orchard. His garden contains 2 acres, his pasture 6 acres, his wood-lot 4 acres, and his orchard enough to make up 18 acres. How many acres does his orchard contain? If he should sell his wood-lot and orchard, how many acres would he have left?

4. I had 17 dollars this morning, but I have since bought a hat for 4 dollars, and a pair of boots for 6 dollars. I have also received 11 dollars, which a friend owed me, and paid a debt of 7 dollars. How much money have I now?

LESSON XIX.

Nineteen wild geese. 19 wild geese. 19 wild geese.

Nineteen. 19. 19. XIX.

A. 1. 10+9?
2. 9+10?
3. 10+*=19?
4. 9+*=19?
5. 14+2+*=19?
6. 4+2+*=19?
7. 19—3—3?
8. 19—3—13?

B. How many more are—
1. 9 than 2?
2. 19 than 2?
3. 19 than 12?
4. 8 than 3?
5. 18 than 3?
6. 18 than 13?

C. 1. 4+3+4+5?
2. 8+3+*=19?
3. 5+7+4—8?
4. 8+5+6—7?
5. 6+4+4+4?
6. 4+6+*=19?
7. 9+3+5—*=8?
8. 10+4+4—*=11?
9. 3+3+3+3+3+3?
10. 1+3+3+3+3+3?

D. 1. One day, Edward and Susan saw a flock of wild geese, which contained just 19. Some sportsmen shot 4 of them, which so frightened the rest, that 6 of them flew towards the east, and the remainder towards the west. Another party of hunters seeing those which were flying towards the west, shot 2 of them, when the rest flew towards the east, and joined that part of the flock which had first flown in that direction. How large a flock was there then?

2. Frank has money enough to buy a pencil for 3 cents, a pen for 6 cents, some ink for 4 cents, and an inkstand for 5 cents. How much money has he?

3. Frank's sister has money enough to buy a pen, and inkstand like Frank's, and 5 cents worth of paper. How many cents has she?

4. Walter and Reuben had each 12 cents. But Walter earned 5 cents by doing an errand, and Reuben spent 5 cents for confectionery. How many cents had each of the boys then? How many more had Walter than Reuben?

LESSON XX.

Twenty soldiers. 20 soldiers. *20 soldiers.*

Twenty. 20. *20.* XX.

A. 1. 10+10? 3. 20=* tens?
2. 10+*=20? 4. 20—10?

B. 1. 3+*=10? 4. 1+*=10?
2. 3+*=20? 5. 1+*=20?
3. 13+*=20? 6. 11+*=20?

C. 1. 3+5+7+4? 4. 2+1+4+9+4?
2. 2+9+4+5? 5. 4+9+2+2+2?
3. 6+7+3+5? 6. 2+4+4+4+4?

7. $2+2+2+2+2+2+2+2+2+2$?
8. $1+2+2+2+2+2+2+2+2+2$?
9. $3+3+3+3+3+3$?
10. $1+3+3+3+3+3+3$?
11. $2+3+3+3+3+3+3$?
12. $20-2-2-2-2-2-2-2-2+2+2$?
13. $19-2-2-2-2-2-2-2-2-2$?
14. $20-3-3-3-3-3-3$?
15. $19-3-3-3-3-3-3$?
16. $18-3-3-3-3-3-3$?

D. 1. Little Willie had 20 toy-soldiers. He placed 3 in front for officers, and then arranged the rest in 3 rows, placing 7 in the first row, 6 in the second, and the rest in the third. How many did he place in the third row?

2. Arthur was telling his mother about the boys who went to his school. He said that 4 of them had neither hoops nor kites, that 6 had kites only, that 7 had hoops only, and that 3, including himself, had both hoops and kites, and this comprised all the boys in the school. How many boys were there in the school?

3. Isaac and Francis were playing ball with Augustus and Reuben. Isaac batted the ball 11 times, and Francis batted the ball 9 times. Augustus batted it 10 times, and Reuben 7 times. How many times did Isaac and Francis bat it? How many times did Augustus and Reuben bat it? Isaac caught the ball 9 times, and Francis caught it 7 times. Augustus caught it 9 times, and Reuben caught it 10 times.

How many more times did Augustus and Reuben catch it than Isaac and Francis?

4. Three idle boys, Thomas, Joseph, and Samuel, were disputing about their examples. Samuel said he performed 6 examples on Monday, 3 on Tuesday, and 5 on Wednesday. Joseph said he performed 3 on Monday, 5 on Tuesday, and 6 on Wednesday. Thomas said he performed 6 on Monday, 3 on Tuesday, and 5 on Wednesday. Each thought he had performed more than either of the others; so they quarreled about it. Now, can you tell who had done the most? William, who was an industrious boy, performed 20 examples on Monday. How many more than Samuel did he perform on that day? How many more than Joseph? How many more than Thomas? How many more than each of the others performed in three days?

5. Susan and Mary had each 9 oranges. Susan gave 5 of hers to Mary, and Mary ate 2. They then put what they had left together, intending to keep them until the next week; but before that time 4 of them had spoiled. They then so divided the good ones among them, that Mary had 6. How many had Susan?

6. Mr. Wheelock had 6 dollars, but he has since received 9 dollars, spent 10 dollars for broad-cloth, received an old debt of 2 dollars, found 2 dollars, received 7 dollars for work, paid 8 dollars for a barrel of flour, 2 dollars for a barrel of apples, and lost 6 dollars. How many dollars has he now?

LESSON XXI.

2 tens=twenty, and is written 20, or *20*.

3 tens=thirty, and is written 30, or *30*.

4 tens=forty, and is written 40, or *40*.

5 tens=fifty, and is written 50, or *50*.

6 tens=sixty, and is written 60, or *60*.

7 tens=seventy, and is written 70, or *70*.

8 tens=eighty, and is written 80, or *80*.

9 tens=ninety, and is written 90, or *90*.

10 tens=one hundred, and is written 100, or *10*

B. How many tens are there—

1. In 50?
2. In 80?
3. In 30?
4. In 70?
5. In 20?
6. In 60?
7. In 90?
8. In 40?
9. In 100?

C. What number is equal to each of the followin

1. 3 tens?
2. 9 tens?
3. 10 tens?
4. 4 tens?
5. 8 tens?
6. 7 tens?
7. 6 tens?
8. 2 tens?
9. 5 tens?

D. How will you write each of the following nu
bers in figures?

1. Forty?
2. Eighty?
3. Twenty?
4. Ninety?
5. Seventy?
6. Fifty?
7. Thirty?
8. Sixty?
9. One hundred?

E. 1. 4 tens + 5 tens? Then 40 + 50?
2. 4 tens + 4 tens? Then 40 + 40?
3. 7 tens + 3 tens? Then 70 + 30?
4. 6 tens + * tens = 9 tens? Then 60 + * = 90?
5. 4 tens + * tens = 10 tens? Then 40 + * = 100?
6. 2 tens + * tens = 6 tens? Then 20 + * = 60?
7. 8 tens — 5 tens? Then 80 — 50?
8. 4 tens — 3 tens? Then 40 — 30?
9. 10 tens — 3 tens? Then 100 — 30?

F. 1. 40+30+20?
2. 50+20+30?
3. 20+30+30?
4. 20+40+20?
5. 40+20+30?
6. 30+30+40?
7. 30+30+40—50?
8. 20+20+20+20+20—70?
9. 100—20—20—20—20+50?
10. 90—60—20+*=60?
11. 20+30+40—*=70?
12. 100—30—30—20+*=80?

G. 1. A man gave 20 cents for Harpers' Magazine, 20 for Putnam's, and 50 for the North American Review. How many cents did he pay for all?

2. A provision-dealer had 20 bushels of potatoes. He afterwards bought 20 bushels of one man, 30 of another, and 30 of another, and then sold 40 bushels to one man, and 30 to another. How many bushels had he left?

3. A farmer, who owned 90 sheep, kept 40 in one pasture, 30 in another, and the rest in another. How many did he keep in the last pasture? He drove 10 sheep from the second pasture

into the first, and 20 from the second into the third. How were then in each pasture?

4. Sarah found 20 walnuts under one tree, and 30 und other, while Lydia found 30 under one tree, and 20 under an How many did each find? How many did both find?

5. Sarah gave 20 nuts to Jane, and Lydia gave her 10. many did both give her? How many had each left? How had the three girls?

6. George has 20 cents, Williams has 20, and Edward l more than George and William together. How many has Ed How many have all the boys? They agreed to put their together and purchase some articles with it. They bought paper for 20 cents, some pencils for 20 cents, and some pc 10 cents, and a pretty story-book for the rest of their n How much did the story-book cost?

LESSON XXII.

A. A unit is a single thing, or one.

2 tens + 1 unit = twenty-one = 21.
2 tens + 2 units = twenty-two = 22.
2 tens + 9 units = twenty-nine = 29.
3 tens + 1 unit = thirty-one = 31.
8 tens + 7 units = eighty-seven = 87.
9 tens + 9 units = ninety-nine = 99.

B. Count from *twenty* to *one hundred*, thu
Twenty-one, twenty-two, twenty-three, &c., &c.

What is the value of—

1. 20 + 8?
2. 40 + 6?
3. 80 + 7?
4. 90 + 3?
5. 60 + 9?
6. 20 + 5?

C. How many tens and units are there —

1. In 64?
2. In 37?
3. In 73?
4. In 28?
5. In 50?
6. In 07?
7. In 23?
8. In 19?
9. In 91?

Explanation.—The figure which represents the units of a number is called the **units' figure**, and that which represents the tens is called the **tens' figure.**

Point out the tens' figure, and also the units' figure, in the numbers written under letter C.

The position, or place occupied by the unit's figure, is called the *units' place*, and that occupied by the tens' figure is called the *tens' place*. The tens' place is always just at the left of the units.

A period, called the *decimal point*, is often used to aid in determining the place of figures; the first place at the left of the point being the units' place, the second at the left being the tens' place, and the third the hundreds'. When the point is not written, it is understood to belong at the right of the given number, thus making the right-hand figure the units' figure.

E. Write each of the following numbers:—

1. Seventy-nine.
2. Twenty-four.
3. Fifty-seven.
4. Sixty-nine.
5. Eighty-six.
6. One hundred.

F. 1. $4 + 2 + 2$?

2. $24 + 2 + 2$?

3. $34 + 2 + 2$?

4. $4 + 72 + 2$?

5. $5 + 3 + 2$?

6. $65 + 3 + 2$?

7. $95 + 3 + 2$?

8. $85 + 3 + 2$?

9. $3 + 4 + * = 9$?

10. $3 + 4 + * = 29$?

11. $3 + 24 + * = 29$?

12. $3 + 74 + * = 79$?

13. $2 + 4 + * = 10$?

14. $2 + 4 + * = 40$?

15. $2 + 4 + * = 70$?

16. $2 + 4 + * = 100$?

G. 1. $2 + 5 + 3 + 3 + 2 + 5 + 3 + 5 + 2 + 3$?

2. $2 + 7 + 1 + 2 + 7 + 1 + 2 + 7 + 1 + 2$?

3. 3 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3?

4. 53 + 3 + 4 + 3 + 3 + 4 + 3 + 3 + 4 + 3?

5. 24 + 2 + 4 + 1 + 7 + 2 + 4 + 6 + 2 + 5?

6. 35 + 8 + 2 + 3 + 7 + 1 + 9 + 2 + 4 + 2?

7. 42 + 8 + 5 + 3 + 2 + 4 + 6 + 2 + 7 + 1?

8. 57 + 3 + 1 + 3 + 5 + 1 + 2 + 3 + 4 + 1?

9. 14 + 6 + 4 + 6 + 3 + 7 + 5 + 5 + 4 + 4?

10. 31 + 2 + 4 + 3 + 5 + 5 + 8 + 2 + 3 + 4?

H. 1. 9 — 3 — 4?

2. 29 — 3 — 4?

3. 99 — 3 — 4?

4. 10 — 2 — 5?

5. 40 — 2 — 5?

6. 50 — 2 — 45?

I. 1. 70 — 4 — 3 — 3 — 4 — 3 — 3 — 4?

2. 100 — 3 — 5 — 2 — 3 — 5 — 2 — 3?

3. 80 — 2 — 4 — 4 — 2 — 4 — 4 — 2?

4. 40 — 8 — 2 — 5 — 5 — 2 — 4 — 3?

5. 70 — 6 — 4 — 2 — 8 — 3 — 5 — 2?

6. 100 — 3 — 4 — 3 — 10 — 5 — 5 — 8?

7. 25 + 3 + 2 + 8 — 5 — 3 — 6 — 4?

8. 83 + 7 + 2 + 8 — 3 — 7 — 5 — 5

9. 63 + 4 + 2 + 1 + 2 + 4 — 6 — 5?

10. 90 — 8 — 2 — 3 — 7 — 9 + 6 + 8?

J. 1. One day, George was reckoning up his money. He sa father gave him 25 cents at one time, and 5 at another; th mother gave him 3 cents at one time, and 7 at another; and his uncle Rufus gave him 6 cents at one time, and 8 cents a other. How many cents had he?

2. Samuel thinks that the sum of 23 + 4 + 3 + 6 + 4 + 4 +

49, while William thinks it is 48, and Lydia thinks it is 47. How much do you think it is?

3. Sarah had 50 cents; she spent 10 cents for ribbon, 4 cents for sewing-silk, and 5 cents for needles. How many cents had she left?

4. As Eliza was picking up shells on the sea-shore, she found 15 in one place, 5 in another, 8 in another, 7 in another, and enough to make up 40 in another. How many did she find in the last place?

5. Erastus and Edwin played "odd or even," beginning their game with 20 grains of corn a-piece. The first time Erastus won 3 grains from Edwin, the second time he won 3 grains, and the third time he won 4. How many had each boy then?

6. A party of hunters, on counting their game, found that they had shot 28 pigeons, 7 partridges, 20 quails, 3 woodcocks, and 3 snipes. How many birds had they shot in all?

7. Laura found 44 blackberries in one place, 6 in another, 2 in another, 8 in another, and 9 in another, when, feeling tired, she sat down to rest. She ate 9, and put 4 in a hole for a squirrel to eat, and threw 6 to some birds. She then found 20, and started for home, but on her way she unfortunately lost 8. How many had she to carry home?

8. Mr. Day went out to pay some debts that he owed, and to collect some money that was due him, taking with him 30 dollars. He paid 7 dollars to a shoemaker, 3 dollars to a laborer, and 5 dollars to a hatter. He then received 5 dollars from Mr. Baker, 30 dollars from Mr Smith, and 8 dollars from Mr. Sumner, after which he paid Mr. Gay 6 dollars for groceries, and 2 dollars for cloth, and Dr. Fogg 7 dollars for services as a physician. How much money had he left?

LESSON XXIII.

A. 1. 9 + 8?
2. 19 + 8?
3. 49 + 8?
4. 6 + 9?
5. 36 + 9?
6. 6 + 89?
7. 4 + 7?
8. 14 + 7?
9. 4 + 37?
10. 4 + 67?

B. 1. 6 + 8 + 9?
2. 16 + 8 + 9?
3. 66 + 8 + 9?
4. 9 + 7 + 8?
5. 89 + 7 + 8?
6. 9 + 77 + 8?
7. 6 + * = 14?
8. 6 + * = 34?
9. 46 + * = 54?
10. 8 + * = 11?
11. 18 + * = 21?
12. 3 + * = 61?

C. 1. 13 — 8?
2. 23 — 8?
3. 93 — 8?
✓4. 14 — 6?
5. 44 — 6?
6. 94 — 6?
7. 11 — 5?
8. 81 — 5?
9. 41 — 5?
10. 91 — 5?

TO THE TEACHER. — Vary and extend the preceding exercise the scholars appreciate the connexion between 9 + 8, 15 + 8, 29 + 8, 12 + 9, 23 + 9, 83 + 9, &c., and understand fully that as 9 + 8 = 1 7 more than 10, so 49 + 8 = 7 more than 50, or 57; that as 13 — 9 so 23 — 9 = 14, 83 — 9 = 74, &c. The great objects to be aimed a accuracy and promptness, the latter being scarcely less important the former.

D. 1. 48 + 4?

2. 37 + 6?

3. 29 + 7?

4. 53 + 8?

5. 74 + 7?

6. 23 + 9 + 5 + 8?

7. 27 + 6 + 8 + 5?

8. 34 + 8 + 9 — 7?

9. 26 + 3 + 5 — 6?

10. 25 + 8 + 4 — 9?

11. 29 + 7 + 9 + 4?

12. 73 + 9 + 6 + 9?

13. 28 + 5 + 8 + 7 + 4?

14. 49 + 7 + 6 + 8?

15. 67 + 8 + 4 + 9?

16. 57 + 8 + 4 + 6?

17. 57 + 6 — 2 — 8?

18. 67 + 8 — 9 — 6?

19. 33 + 8 + 4 + 9 + 7 + 6 + 5 + 3?

20. 56 + 6 + 9 + 3 + 8 + 5 + 4 + 6?

21. 45 + 8 + 9 + 7 + 4 + 9 + 6 + 5?

22. 38 + 6 + 7 + 7 + 5 + 8 + 9 + 8?

E. Find the sum of the following columns:—

1	2	3	4	5
6	2	9	3	3
7	7	5	6	9
9	4	9	6	6
4	8	7	4	2
3	5	6	5	7
8	9	9	8	5
5	3	3	7	4
2	3	9	2	4
6	6	5	3	8
4	7	9	6	3
3	9	4	7	9
—	—	—	—	—

To THE TEACHER.—It will be a valuable exercise for the p count by twos, threes, fours, &c., *i. e.*, to call the results obtai successive additions of the same number to itself, or some othe ber, till all possible combinations are exhausted.

Thus, in adding threes, we shall exhaust the varieties of comb by beginning thus: THREE, SIX, NINE, TWELVE, &c.; TWO, FIVE, ELEVEN, &c.; ONE, FOUR, SEVEN, TEN, &c. A similar course taken in subtraction.

F. 1. Henry had 42 cents. He earned 9 cents by doing e 5 cents by holding a gentleman's horse, and 8 cents by deli a letter, after which he spent 7 cents. How many cents had h

2. A newsboy bought 8 copies of the Boston Post, 9 Atlas, 10 of the Traveller, 7 of the Bee, 10 of the Journal, of the Transcript. How many papers did he buy in all? I all but 7 of them. How many did he sell?

3. A trader bought 8 yards of cloth, for which he paid lars; 6 yards for which he paid 9 dollars, 9 yards for wh paid 8 dollars, and 6 yards for which he paid 8 dollars. many yards of cloth did he buy in all? How many doll he pay for it?

4. A man bought a horse for 63 dollars, and was obliged him for 8 dollars less than he cost him. For how much sell him?

5. A farmer who had 33 bushels of corn, sold 6 bushel dollars. How many bushels had he left?

6. Mr. Adams owned 46 acres of land, and bought eno make up 54 acres. How many did he buy?

7. Sarah had 32 roses. She gave 9 to one of her comp 8 to another, and, when she had given some to another, s 7 left. How many did she give to the last?

8. A trader bought a lot of grain for 54 dollars, and it c 8 dollars more to have it carried to his store. For how must he sell it to gain 9 dollars?

9. Mr. Edwards sold a colt for 57 dollars, a sheep for 8 dollars, a calf for 5 dollars, and a cow for 30 dollars, and in part payment received a horse worth 98 dollars. How much still remained due?

10. Mr. Boyden and Mr. Manchester each bought a yoke of oxen. Mr. Boyden gave in payment for his oxen, a cart worth 47 dollars, 1 ten-dollar bill, 1 five-dollar bill, 1 three-dollar bill, 7 one-dollar bills, and 9 silver dollars. Mr. Manchester gave in payment for his oxen, a cow worth 30 dollars, a double eagle worth 20 dollars, an eagle worth 10 dollars, a half-eagle worth 5 dollars, an ox-yoke worth 10 dollars, and 9 dollars worth of hay. Which paid the most for his oxen, and how much the most?

LESSON XXIV.

A. 1. 30 + 50?
2. 32 + 50?
3. 39 + 50?
4. 20 + 60?
5. 25 + 60?
6. 20 + 68?
7. 40 + 20 + 20?
8. 43 + 20 + 20?
9. 40 + 28 + 20?
10. 30 + 20 + 40?
11. 38 + 20 + 40?
12. 30 + 20 + 47?

B. 1. How many are 24 + 67?

Solution.—24 and 60 are 84, and 7 are 91.

2. 63 + 29?
3. 26 + 55?
4. 37 + 48?
5. 24 + 37?
6. 73 + 19?
7. 28 + 53?
8. 37 + 67?
9. 29 + 29?
11. 27 + 58 + 12?
12. 33 + 47 + 16?
13. 24 + 29 + 47?
14. 24 + 29 + 37?
15. 27 + 27 + 27?
16. 11 + 46 + 25?
17. 34 + 26 + 27?
18. 16 + 17 + 19?

C. 1. 80 — 20?
2. 86 — 20?
3. 60 — 30?
4. 67 — 30?
5. 70 — 40?
6. 77 — 40?
7. 90 — 80?
8. 97 — 80?
9. 80 — 40?
10. 86 — 40?
11. 50 — 30?
12. 53 — 30

D. 1. 68 — 26?

Solution.—68 minus 20 are 48, minus 6 are 42.

2. 43 — 17?
3. 92 — 67?
4. 83 — 48?
5. 61 — 23?
6. 56 — 19?
7. 81 — 28?
8. 52 — 27?
9. 48 — 29?
10. 97 — 58?

E. 1. 63 + 37 — 82?
2. 48 + 35 — 27?
3. 24 + 67 — 19?
4. 64 + 36 — 48?
5. 25 + 39 — 42?
6. 27 + 64 — 18?

F. 1. Joseph bought a "First Book of Arithmetic" for cents, and a slate for 13 cents. How much did he pay for bot

2. Martha's mother gave her 75 cents with which to purch school-books and paper. She bought a Primary Geography 37 cents, a Spelling Book for 17 cents, and spent the rest of h money for paper. How much did she spend for paper?

3. A farmer sold a horse for 93 dollars, which was 26 doll more than he gave for him. How much did he give for him?

4. A horse dealer bought a horse for 54 dollars, and after pa ing 17 dollars for keeping him, he sold him for 96 dollars. H much did he gain by the transaction?

5. A man bought a sleigh for 24 dollars. He paid 9 dollars for painting and repairing it, and then gave it and 18 dollars in money for another sleigh. How much did the second sleigh cost him?

6. From a cask containing 64 gallons of oil, 18 gallons were drawn out at one time, and 25 at another, after which 17 gallons were put in. How many gallons were then in the cask?

7. There were 18 sheep in one flock, 27 in another, and 39 in another; but at night they were all put into the fold. How many were there in the fold? The next day, 23 were driven to one pasture, 26 to another, and the rest to another. How many were driven to the last pasture?

8. Ralph shot 27 pigeons, 15 partridges, 14 woodcocks, and as many quails as there were partridges and woodcocks together. How many quails did he shoot? How many birds in all?

9. Mr. Thompson owes 13 dollars to Mr. Baker, 9 dollars more to Mr. Ellis than to Mr. Baker, and as much to Mr. French as he owes to Mr. Thompson and Mr. Ellis together. How much does he owe to each, and how much to all?

10. Mr. Talbot bought a large lot of apples. His son George asked how much they cost him, to which he replied: "I paid 17 dollars in silver, 25 in gold, and 13 dollars more in bank bills than in silver and gold together. Now, if you will tell me what they cost, I will give you the difference between their cost and 100 dollars." George answered correctly. What was his answer, and how much money did his father give him?

LESSON XXV.

A. The numbers above one hundred are counted thus:—

One hundred one, one hundred two, one hundred three, &c., to *one hundred ninety-eight, one hundred ninety-nine, two hundred, two hundred one, two hundred two, two hundred three, &c.,* to *ten hundred,* which is generally called *one thousand, ten hundred one,* or *one thousand one, ten hundred two,* or *one thousand two,* &c., to *eleven hundred one,* or *one thousand one hundred one,* &c., to *nineteen hundred ninety-nine,* or *one thousand nine hundred ninety-nine, twenty hundred,* or *two thousand.*

Ten hundred, or one thousand is written 1000. Twenty hundred, or two thousand, is written 2000. Thirty hundred, or three thousand, is written 3000. Ninety hundred, or nine thousand, is written 9000.

The following exercises suggest the manner of reading and writing numbers above one hundred:—

$100 + 2 = 102$.
$400 + 9 = 409$.
$100 + 10 = 110$
$100 + 11 = 111$.
$300 + 12 = 312$.
$600 + 20 = 620$.
$100 + 29 = 129$.
$1000 + 1 = 1001$.
$1000 + 4 = 1004$.

$1000 + 10 = 1010$.
$1100 + 3 = 1103$.
$1000 + 28 = 1028$.
$1100 + 11 = 1111$.
$1100 + 17 = 1117$.
$1000 + 117 = 1117$.
$3200 + 20 = 3220$.
$4200 + 34 = 4234$.
$4000 + 234 = 4234$.

B. Read the following numbers: —

1. 427
2. 168
3. 250
4. 630
5. 514
6. 2107
7. 1526
8. 3794
9. 1728
10. 6006
11. 2224
12. 1347

C. Write each of the following in figures: —

1. Three hundred twenty-seven.
2. Eight hundred four.
3. Seventeen hundred twenty-eight.
4. Forty-six hundred thirty-six.
5. Four thousand six hundred thirty-six.
6. Twenty-six hundred six.

To the Teacher. — For other exercises in Notation and Numeration, see Arithmetic and its Applications.

LESSON XXVI.

Addition.

A. All such questions as "How many are 6 + 9 + 7?" "4+8+9?" &c., are questions in Addition. We are required in the first to add 6, 9, and 7 together; and in the second, to add 4, 8, and 9. It is obvious that in each, we are required to find a number equal in value to all the given numbers. Thus, in the first question, we are required to find a number equal in value to 6+9+7.

Addition is a process by which we find a number equal in value to several given numbers.

The number thus found is called the SUM or AMOUNT of the given numbers. Thus the sum of 6, 9, and 7 is 22, for $6+9+7=22$.

B. When *writing* large numbers for addition, we place them in a column, so that the figures of the same denomination shall come under each other, *i. e.*, so that units shall come under units, tens under tens, &c. We then begin at the right hand, and add the columns separately, as in the following examples: —

1. What is the sum of 723 + 896 + 589 + 967?

SOLUTION. — Writing the numbers as opposite, we first add the units' column, $7+9+6+3=25$ units $=2$ tens and 5 units. Writing 5 units, and adding 2 tens to the tens' column, we have $2+6+8+9+2=27$ tens $=2$ hundreds, and 7 tens. Writing 7 tens, and adding 2 hundreds to the hundreds' column, we have $2+9+5+8+7=31$ hundreds, which, being the sum of the last column, we write. The answer, therefore, is 3175.

$$\begin{array}{r} 723 \\ 896 \\ 589 \\ 967 \\ \hline 3175 \end{array}$$

To test the correctness of the work, examine it carefully to see if any error can be detected. Or, add the numbers again, beginning at the top of the column.

TO THE TEACHER.—The design of this work renders it impracticable to give further illustrations here, but the Teacher can readily supply them if they are needed by the class. (See ARITHMETIC AND ITS APPLICATIONS, Sect. IV.)

C. Add the following: —

1	2	3	4	5
249	417	827	296	843
372	286	613	178	248
648	439	428	859	537
376	579	854	438	429

6. 425 + 487 + 569 + 837 + 694?
7. 854 + 308 + 560 + 746 + 593?
8. 672 + 481 + 326 + 425 + 519?
9. 243 + 495 + 826 + 324 + 476?
10. 627 + 756 + 434 + 874 + 999?

LESSON XXVII.

Subtraction.

A. Such questions as "4 from 9?" "12 — 6?" "How many more are 17 than 8?" &c., are questions in SUBTRACTION.

We are required, in the first, to subtract 4 from 9; in the second, to subtract 6 from 12; and in the third, to subtract 8 from 17.

Subtraction is a process by which we find the difference between two numbers, or the excess of one number over another.

The larger of the two given numbers is called the MINUEND, the smaller, or one to be subtracted, is called the SUBTRAHEND, and the answer is called the DIFFERENCE, or REMAINDER.

B. We write large numbers for Subtraction, so that figures of the same denomination shall come under each other, and subtract as illustrated in the following examples: —

1. How much is 8436 — 6122?

SOLUTION.—Writing the numbers as opposite, we have 2 units from 6 units leave 4 units; 2 tens from 3 tens leave 1 ten; 1 hundred from 4 hundreds leaves 3 hundreds; 6 thousands from 8 thousands leaves 2 thousands. Therefore, the answer is 2 thousands, 3 hundreds, 1 ten, and 4 units, or 2314.

$$\begin{array}{r}8436\\6122\\\hline 2314\end{array}$$

In the same manner perform the following:—

2. 4893 — 1231?
3. 5987 — 3125?
4. 8958 — 6713?
5. 4867 — 1614?
6. 9818 — 2106?
7. 6985 — 1401?

C. If a figure of the subtrahend is larger than the corresponding figure of the minuend, we take one of the next higher denomination of the minuend, and reduce (*i. e.*, change) it to the required denomination, as in the following example:—

1. How much is 947 — 458?

Solution.—As we cannot subtract 8 units from 7 units, we take one of the 4 tens (leaving 3 tens), and reduce to its value in units: 1 ten = 10 units, which, added to the 7 units, gives 17 units: 17 units — 8 units = 9 units.

8,13,17 = { minuend changed in form to show the reduction.
9 4 7 = minuend.
4 5 8 = subtrahend.
4 8 9 = quotient.

As we cannot take 5 tens from 3 tens, we take one of the 9 hundreds, leaving 8 hundreds, and reduce it to its value in tens: 1 hundred = 10 tens, which, added to the 3 tens = 13 tens; 13 tens — 5 tens = 8 tens.

4 hundreds from 8 hundreds leave 4 hundreds. Therefore, 947 — 458 = 4 hundreds, 8 tens, and 9 units, or 489.

To prove the correctness of the answer, add the subtrahend and remainder together; if their sum is equal to the minuend, the work is correct; if not, there is an error in the subtraction or the addition, and the work should be re-examined to detect it.

To the Teacher.—For more full illustrations, see Arithmetic and its Applications.

2. 4864 — 2579?

3. 8149 — 3463?

4. 2769 — 1487?

5. 2144 — 1397?

6. 8482 — 3586?

7. 4374 — 5856?

8. 5426 — 3987?

9. 9943 — 4399?

10. 9333 — 8888?

11. 4634 — 2359?

12. 9257 — 4328?

13. 8642 — 5853?

LESSON XXVIII.

The method of writing numbers by figures is called the *Arabic Method.*

There is a method of expressing numbers by letters, called the *Roman Method.* The letter I stands for one, V for five, X for ten, L for fifty, C for one hundred, D for five hundred, and M for one thousand.

If a letter is repeated, it indicates that the number for which it stands is repeated. Thus: I stands for one, II for two, III for three, X for ten, XX for twenty, XXX for thirty, CC for two hundred, &c., &c.

If a letter representing one number stands before a letter, representing a larger number, the value of the former is subtracted from the value of the latter. Thus: IV = 1 from 5 = 4, IX = 1 from 10 = 9, XL = 10 from 50 = 40, XC = 10 from 100 = 90, &c.

If a letter representing one number stand before a letter representing a smaller number, the value of the former is to be added to the value of the latter. Thus: VI = 5 + 1 = 6, XI = 10 + 1 = 11, XV = 10 + 5 = 15, &c. CX = 100 + 10 = 110. Hence—

I = 1
II = 2
III = 3
IV = 4
V = 5
VI = 6
VII = 7
VIII = 8
IX = 9
X = 10
XI = 11
XII = 12
XIII = 13
XIV = 14
XV = 15
XVI = 16
XVII = 17
XVIII = 18
XIX = 19
XX = 20
XXI = 21
XXIV = 24
XXV = 25
XXX = 30
XXXIX = 39
XLIV = 44
LXX = 70
LXXXIX = 89
XC = 90
CXXXIX = 139

LESSON XXIX.

Tables of Moneys, Weights, and Measures.

A. The money we use is called UNITED STATES or FEDERAL MONEY.

TABLE OF UNITED STATES MONEY.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

The coins of the United States are: the cent, the three-cent piece, the half-dime, worth 5 cents; the dime, worth 10 cents; the quarter-dollar, worth 25 cents; the half-dollar, worth 50 cents; the dollar, worth 100 cents; the three-dollar piece; the eagle, worth 10 dollars; the double-eagle, worth twenty dollars; the half-eagle, worth five dollars; the quarter-eagle, worth two and a half dollars, and the fifty-dollar piece.

---

* In reciting these tables, let the pupils say "equal" in place of "make one," the phrase often used.

The character $ placed at the left of figures, shows that they represent dollars, or values in United States Money. The dollars are always placed at the left of the decimal point (See Lesson XXV), and the cents and mills at the right.

Thus, to express 14 dollars 38 cents, we should write 14 at the left of the decimal point, and 38 at the right of it. Thus: $14.38.

ILLUSTRATION.—$ 8.27 = 8 dollars, 27 cents.
$15.06 = 15 dollars, 06 cents.
$2.327 = 2 dollars, 32 cents, 7 mills.

Read the following: —

1. $4.28.
2. $5.27.
3. $82.36.
4. $75.07.
5. $40.03.
6. $28.79.

B. The money used in England is called ENGLISH or STERLING MONEY.

TABLE OF STERLING MONEY.

FULL TABLE.	ABBREVIATED TABLE.
4 farthings = 1 penny.	4 gr. = 1d.
12 pence = 1 shilling.	12 d. = 1s.
20 shillings = 1 pound.	20 s. = 1£.

The English pound is worth *about* $4.84.

The English shilling is worth *about* a quarter of a dollar, and the English penny is worth *about* 2 cents. The term shilling is sometimes used in New York, New England, and some other States of the Union, but it does not mean an English shilling. A New York shilling is worth just 12½ cents. A New England shilling is worth just 16⅔ cents. The ninepence of New England is the same as the shilling of New York.

C. Iron, flour, sugar, wool, coal, and almost all articles except gold, silver, and jewels, are weighed by AVOIRDUPOIS WEIGHT.

FULL TABLE.	ABBREVIATED TABLE.
16 drams = 1 ounce.	16 dr. = 1 oz.
16 ounces = 1 pound.	16 oz. = 1 lb.
25 pounds = 1 quarter.	25 lbs. = 1 qr.
4 quarters = 1 hundred weight.	4 qrs. = 1 cwt.
20 hundred weight = 1 ton.	20 cwt. = 1 T.

NOTE.—Formerly the quarter was reckoned at 28 lbs., the hundred-weight at 112 pounds, and the ton at 2240 lbs., and they are so reckoned at the present time in Great Britain, and at the United States Custom Houses. Merchants usually reckon them as given in the table.

D. Gold, Silver, and precious stones are weighed by TROY WEIGHT.

FULL TABLE.	ABBREVIATED TABLE.
24 grains = 1 pennyweight.	24 gr. = 1 dwt.
20 pennyweights = 1 ounce.	20 dwt. = 1 oz.
12 ounces = 1 pound.	12 oz. = 1 lb.

E. APOTHECARIES' WEIGHT is used in compounding or mixing medicines, but they are sold by Avoirdupois weight.

FULL TABLE.	ABBREVIATED TABLE.
20 grains = 1 scruple.	20 gr. = 1 ℈.
8 scruples = 1 dram.	8 ℈ = 1 ʒ.
8 drams = 1 ounce.	8 ʒ = 1 ℥.
12 ounces = 1 pound.	12 ℥ = 1 ℔.

F. COMPARISON OF AVOIRDUPOIS, TROY, AND APOTHECARIES' WEIGHT.

A pound Avoirdupois is heavier than a pound Troy, but an ounce Avoirdupois is not so heavy as an ounce Troy.

Their relative weights may be seen in the following table of comparison, which expresses the value of each in grains Troy:—

1 lb. Avoirdupois = 7000 grains Troy.
1 lb. Troy = 1 ℔. = 5760 " "
1 oz. Avoirdupois = 437½ " "
1 oz. Troy = 1℥ = 480 " "
1 dr. Avoirdupois = $27\frac{11}{32}$ " "
1 ʒ Troy = 60 " "
1 ℈ " = 20 " "
1 dwt. = 24 " "
1 gr. Apothecaries = 1 " "

follows, then, that —

144 lbs. Avoirdupois = 175 Troy.

192 oz. " = 175 oz. Troy.

1 lb. " = $\frac{175}{144}$ of 1 lb. Troy.

1 oz. " = $\frac{175}{192}$ of 1 oz. Troy.

Long Measure is used for measuring lengths and dis-
s.

ULL TABLE.	ABBREVIATED TABLE.
ıes = 1 inch.	12 l. = 1 in.
ches = 1 foot.	12 in.= 1 ft.
et = 1 yard.	8 ft. = 1 yd.
ards, or eet } =1 rod, or pole.	5½ yds., or 16½ ft. } = 1 rd. or p.
ds = 1 furlong.	40 rds. = 1 fur.
ırlongs = 1 mile.	8 fur. = 1 m.
iles = 1 league.	8 m. = 1 le.

Cloth Measure is used for measuring cloths, silks, &c.

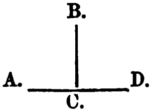
FULL TABLE.	ABBREVIATED TABLE.
2¼ inches = 1 nail.	2¼ in. = 1 na.
4 nails = 1 quarter.	4 na. = 1 qr.
4 quarters = 1 yard.	4 qr. = 1 yd.

Square Measure. — This measure is used in measuring
and all kinds of surfaces.

eliminary Definitions.—An *angle* is the difference in direction
o lines. The point where the lines meet is called the *vertex*
angle.

hen the two angles formed by one straight line meeting
her are equal to each other, they are called *right angles.*

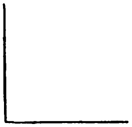
One line is *perpendicular* to another when it makes righ
with it.

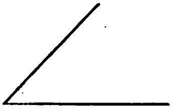


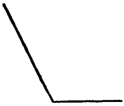
The angle A C B is equal to th
B C D, and hence they are right
Therefore, B C is perpendicular to

An angle greater than a right a
called an *obtuse angle*, and an angle less than a right a
called an *acute angle*.

A RIGHT ANGLE. AN ACUTE ANGLE. AN OBTUSE A







A four-sided figure having all of its angles right angles, i
a *rectangle*.

A rectangle having all of its sides equal, is called a *squ*
square, then, has four equal sides, and four equal angles.

A square foot is a square measuring one foot on every s
square yard is a square measuring a yard on every side,

TABLE OF SQUARE MEASURE.

FULL TABLE.	ABBREVIATED TABLE.
144 square inches = 1 square foot.	144 sq. in. = 1 sq.
9 square feet = 1 square yard.	9 sq. ft. = 1 sq. y
30¼ square yards, or 272¼ square feet, } = 1 sq. rod.	30¼ sq. yds. or 272¼ sq. ft. } =1
40 square rods = 1 rood.	40 sq. rds. = 1 R.
4 roods = 1 acre.	4 R. = 1 A.
640 acres = 1 square mile.	640 A. = 1 sq. m.

J. **CUBIC MEASURE.**—Cubic Measure is used in me
solids.

A solid is a magnitude which has length, breadth, and thi

A cube is a rectangular solid, whose length, breadth, and height, are equal. It may also be defined as a solid bounded by six equal squares.

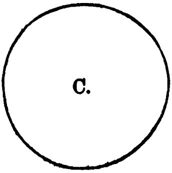
A cube 1 foot long, 1 foot wide, 1 foot high, would be a cubic foot.

A cube 1 yard long, 1 yard high, and 1 yard wide, would be a cubic yard.

FULL TABLE.	ABBREVIATED TABLE.
1728 cubic inches = 1 cubic foot.	1728 cu. in. = 1 cu. ft.
27 cubic feet = 1 cubic yard.	27 cu. ft. = 1 cu. yd.
16 cubic feet = 1 cord foot.	16 cu. ft. = 1 cd. ft.
8 cord feet, or 128 cubic feet } =1 cord wd.	8 cd. ft. or 128 cu. ft. } =1 cd. wd.

K. CIRCULAR OR ANGULAR MEASURE. — Circular or Angular Measure is used to measure angles, and the circumferences of circles.

A circle is a surface bounded by a curved line, which is everywhere equally distant from a point within, called the *centre*. The boundary line is called the *circumference* of the circle.



The figure represents a circle, of which C. is the centre.

The distance from the centre of a circle to the circumference is called the *radius*.

The distance from a point on one side of a circle through the centre to a point on the opposite side is called the *diameter*. Any portion of the circumference is called an *arc*.

Every circumference of a circle, whether large or small, is supposed to contain 360 equal parts, called degrees. Each degree is divided into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds.

A degree may be considered simply as the 360th part of the circumference of the circle considered. Hence its length, as well as that of its subdivisions, must vary with the size of the circle.

FULL TABLE.	ABBREVIATED TABLE.
60 seconds = 1 minute.	60″ = 1′.
60 minutes = 1 degree.	60′ = 1°.
360 degrees = 1 circumference.	360° = 1 circ.

L. **Dry Measure** is used for measuring grain, nuts, salt, &c.

FULL TABLE.	ABBREVIATED TABLE.
2 pints = 1 quart.	2 pts. = 1 qt.
8 quarts = 1 peck.	8 qts. = 1 pk.
4 pecks = 1 bushel.	4 pks. = 1 bu.

The chaldron of 36 bushels is sometimes used in measuring coal. Ch. is the sign for chaldron.

The bushel contains 2150⅖ cubic inches, and the quart contains 67⅕ cubic inches.

M. All kinds of liquids are measured by **Liquid Measure**.

FULL TABLE.	ABBREVIATED TABLE.
4 gills = 1 pint.	4 gls. = 1 pt.
2 pints = 1 quart.	2 pts. = 1 qt.
4 quarts = 1 gallon.	4 qts. = 1 gal.

The hogshead of 63 gallons is used in estimating the contents of reservoirs, or other large bodies of water; but in all other cases the term hogshead is not a definite measure. Casks containing from 50 or 60, to 100 or 200 gallons, are called hogsheads.

A barrel of cider is usually reckoned at 31½ gallons.

The gallon contains 231 cubic inches.

The beer gallon is sometimes used in measuring beer, milk, and ale. It contains 282 cubic inches, and the beer quart contains 70½ cubic inches.

N. COMPARISON OF DRY, LIQUID, AND BEER MEASURE.

1 qt. dry measure = 67⅕ cubic inches.
1 qt. liquid measure = 57¾ cubic inches.
1 qt. beer measure = 70½ cubic inches.

O. TABLE OF TIME.

FULL TABLE.	ABBREVIATED TABLE.
60 seconds = 1 minute.	60 sec. = 1 min.
60 minutes = 1 hour.	60 min. = 1 h.
24 hours = 1 day.	24 h. = 1 d.
7 days = 1 week.	7 d. = 1 wk.
365 days, or 52 weeks, 1¼ days. } = 1 year.	365 d. or 52 wk. 1¼ d. } = 1 y.

To avoid the inconvenience of reckoning ¼ day with each year, every fourth year (called leap year) is reckoned at 366 days, and the others at 365.

The year is divided into 12 months, which differ somewhat in length, as is seen in the following

TABLE OF MONTHS.

January has 31 days.
February has 28 days.*
March has 31 days.
April has 30 days.
May has 31 days.
June has 30 days.
July has 31 days.
August has 31 days.
September has 30 days.
October has 31 days.
November has 30 days.
December has 31 days.

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* Except in leap year, when it has 29.

P. MISCELLANEOUS.

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

A barrel of beef or pork weighs 200 lbs.
A barrel of flour weighs 196 lbs.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.

BOOKS.

A sheet folded in 2 leaves is called a folio.
“ “ “ “ 4 “ “ “ quarto, or 4to.
“ “ “ “ 8 “ “ “ octavo, or 8vo.
“ “ “ “ 12 “ “ “ duodecimo or 12mo.
“ “ “ “ 18 “ “ “ 18mo.

This book is a duodecimo.

Q. FRENCH MEASURES AND WEIGHTS.

The following measures and weights are often referred to in this country, especially in scientific works:

FRENCH LONG MEASURE.

10 millimetres = 1 centimetre.
10 centimetres = 1 decimetre.
10 decimetres = 1 metre.
10 metres = 1 decametre.

10 decametres = 1 hectometre.
10 hectometres = 1 kilometre.
10 kilometres = 1 myriametre.

The metre is regarded as the unit of measure, and equals 39.371 of our inches. It is the twenty-millionth part of the distance measured on the meridian, from one pole to the other.

FRENCH WEIGHTS.

10 milligrammes = 1 centigramme.
10 centigrammes = 1 decigramme.
10 decigrammes = 1 gramme.
10 grammes = 1 decagramme.
10 decagrammes = 1 hectogramme.
10 hectogrammes = 1 kilogramme.
10 kilogrammes = 1 myriagramme.

The gramme is regarded as the unit of this weight, and equals about 15.$\frac{434}{1000}$ grains Troy.

The kilogramme is the weight most frequently used in business transactions, and equals very nearly 2$\frac{1}{5}$ pounds Avoirdupois.

FRENCH MONEY.

10 centimes = 1 decime.
10 decimes = 1 franc.

The franc equals 18$\frac{3}{5}$ cents, and the five-franc piece often seen in the United States, is equal in value to 93 cents.

LESSON XXX.

A. TABLE.

Add 10 twos together.

2 times 1, or once 2 = 2
2 times 2 = 4
2 times 3, or 3 times 2 = 6
2 times 4, or 4 times 2 = 8
2 times 5, or 5 times 2 = 10
2 times 6, or 6 times 2 = 12
2 times 7, or 7 times 2 = 14
2 times 8, or 8 times 2 = 16
2 times 9, or 9 times 2 = 18
2 times 10, or 10 times 2 = 20

B. 1. * times 7 = 14?
2. * times 2 = 14?
3. * times 4 = 8?
4. * times 2 = 12?
5. * times 2 = 18?
6. 4 = * times 2?
7. 20 = * times 2?
8. 16 = * times 2?
9. 10 = * times 5?
10. 6 = * times 2?

C. 1. 4 times * = 8?
2. 2 times * = 8?
3. 2 times * = 10?
4. 2 times * = 6?
5. 2 times * = 14?
6. 16 = 2 times *?
7. 4 = 2 times *?
8. 20 = 2 times *?
9. 12 = 2 times *?
10. 18 = 2 times *?

D. 1. 8 times 2, plus 4 = * times 10?
2. 2 times 5, plus 8 = * times 2?
3. 7 times 2, minus 6 = * times 4?
4. 2 times 9, minus 6 = * times 2?
5. 2 times 4, plus 8 = * times 2?

To the Teacher.—The reasoning processes of the following examples are very important, and should be thoroughly understood by the scholars. Not till, by much drill and many repetitions, they have become perfectly familiar, can they safely be omitted or neglected. Indeed, if the pupil must, in his first exercises, omit either, it is far better to give the reasoning process, and omit the answer, than to omit the process, and give the answer only.

1. 2 pks. = * qts?

Solution. — Since 1 peck = 8 quarts, 2 pecks must equal 2 times 8 quarts, or 16 quarts. Therefore, 2 pks. = 16 qts.

2. 6 qts. = * pts.?
3. 2 wks. = * da.?
4. 2 dimes = * cents?
5. 2 yds. = * qrs.
6. 2 ʒ. = * ℈?
7. 2 sq. yds. = * sq. ft.?

8. How much will 6 apples cost at 2 cents a-piece?

Solution. — Since 1 apple costs 2 cents, 6 apples will cost 6 times 2 cents, or 12 cents. Therefore, 6 apples at 2 cents each will cost 12 cents.

9. How much will 8 books cost at 2 dollars a-piece?

10. How much will 2 hats cost at 5 dollars a-piece?

11. How far will a man walk in 2 hours, if he walks at the rate of 4 miles per hour?

12. How many bushels will 8 boxes hold, if each box holds 2 bushels?

13. How many quarts of berries will George pick in 9 days, if he picks 2 quarts per day?

14. How much will 10 pairs of shoes cost at 2 dollars per pair?

F. 1. 16 ʒ = * ℥?

Solution.—Since 8 drams Apothecaries' equal one ounce, 16 drams must be equal to as many ounces as there are times 8 in 16, which are 2 times. Therefore, 16 ʒ = 2 ℥.

2. 6 ft. = * yds. ?

3. 20 pts. = * qts. ?

4. 14 da. = * wks. ?

5. 16 qts. = * pts. ?

6. 8 qrs. = * yds. ?

7. 16 pts. = * qts. ?

8. How many apples at 2 cents a-piece can be bought fo cents ?

Solution. — If one apple can be bought for 2 cents, as many a can be bought for 12 cents as there are times 2 in 12, which times. Therefore, 6 apples at 2 cents a-piece can be bought fo cents.

9. How many oranges at 3 cents a-piece can be bought 1 cents ?

10. How many shawls, at $7 each, can be bought for $14

11. How many boxes, holding 2 bushels each, will be requ to hold 20 bushels of apples ?

Solution. — If 1 box is required to hold 2 bushels, as many l will be required to hold 20 bushels as there are times 2 in 20. T fore, 10 boxes, each holding 2 bushels, will be required to contai bushels of apples.

12. How many hours will it take a man to walk 14 miles, i walk at the rate of 2 miles per hour ?

13. How many days would it take a man to earn 10 dollar he earned 2 dollars per day ?

14. How many pieces 8 feet in length can be cut from a p of string 16 feet in length ?

LESSON XXXI.

A. Add 10 threes together.

Note.— The pupil should supply the missing part of this and the subsequent tables, by his knowledge of the preceding ones.

TABLE.

3 times 3 = 9.
3 times 4, or 4 times 3 = 12.
3 times 5, or 5 times 3 = 15.
3 times 6, or 6 times 3 = 18.
3 times 7, or 7 times 3 = 21.
3 times 8, or 8 times 3 = 24.
3 times 9, or 9 times 3 = 27.
3 times 10 or 10 times 3 = 30.

B. 1. * times 3 = 15?
2. * times 3 = 6?
3. * times 7 = 21?
4. * times 3 = 30?
5. 9 = * times 3?
6. 12 = * times 4?
7. 14 = * times 7?
8. 24 = * times 8?

C. 1. 6 times * = 18?
2. 3 times * = 21?
3. 3 times * = 27?
4. 8 times * = 16?
5. 12 = 3 times *?
6. 24 = 3 times *?
7. 15 = 5 times *?
8. 30 = 3 times *?

D. 1. 5 times 3, plus 5 = * times 2?
2. 3 times 8, plus 6 = * times 3?
3. 9 times 3, minus 9 = * times 6?
4. 4 times 3, plus 4 = * times 8?
5. 2 times 9, plus 6 = * times 3?

E. 1. 3 wks. = * da.? 4. 8 yds 3 qrs. =

2. 2 bu. 5 pks. = * pks.? 5. 3 pks. 7 qts. =

3. 9 yds. 2 ft. = * ft.? 6. 7 qts. 1 pt. =

7. Francis says that he has money enough to buy 8 c at 9 cents a-piece, and still have 5 cents left. How mu has he?

8. William has 9 three-cent pieces, and 8 cents besid many cents has he in all?

9. Arthur has 3 half-dimes, and 2 three-cent piec much money has he?

10. How many pen-holders, at 3 cents a-piece, can for 15 cents?

11. Willie had 27 cents, which he exchanged for thei three-cent pieces. How many three-cent pieces did he

12. Amelia had 20 very nice apples. She ate 2, a the rest among her playmates, giving 3 to each. A many did she divide them?

13. If Augustus has 37 apples, how many will he hav giving 3 of his companions 7 apples a-piece?

14. Sarah bought 9 spools of thread at 3 cents a- then had money enough left to buy 2 skeins of silk at 8 skein. How much money had she at first?

15. Simon had 42 cents. He gave 10 cents for a wri and 5 for an inkstand, and then exchanged the rest of for three-cent pieces. How many three-cent pieces did

LESSON XXXII.

TABLE.

A. Add 10 fours together.

4 times 4 = 16.

4 times 5, or 5 times 4 = 20.

4 times 6, or 6 times 4 = 24.

4 times 7, or 7 times 4 = 28.

4 times 8, or 8 times 4 = 32.

4 times 9, or 9 times 4 = 36.

4 times 10, or 10 times 4 = 40.

B. **Explanations and Definitions.**—To multiply a number by 4, is the same as to find 4 times that number; to multiply a number by 7 is the same as to find 7 times that number, &c., &c. Thus, to multiply 6 by 4 is the same as to find 4 times 6, which is 24.

5 multiplied by 3 = 3 times 5 = 15.

8 multiplied by 4 = 4 times 8 = 32.

Multiplication, then, is the process of finding any number of times a given number.

The number to be taken some number of times is called the MULTIPLICAND; the number showing how many times it is to be taken is called the MULTIPLIER; the answer is called the PRODUCT.

Thus, in "8 times 3 = 24," 8 is the multiplier, 3 the multiplicand, and 24 is the product,

Name the multiplier, multiplicand, and product in each of the following examples:—

1. 9 times 4 = 36.
2. 3 times 8 = 24.
3. 7 times 4 = 28.
4. 4 times 6 = 24.
5. 3 times 3 = 9.
6. 4 times 4 = 16.

The multiplier and multiplicand are called FACTORS of the product.

Thus, in 9 times 4 = 36, 9 and 4 are factors of 36.

Name the factors in the above examples.

Two oblique lines crossing thus, ×, form the sign of multi cation. It may be read either as "times" or as "multiplied b

Thus, "6 × 3 = 18" may be read either as "6 times 3 = 18, "6 multiplied by 3 = 18."

To the Teacher. — It will probably be well to have the pupil first read the sign of multiplication as though written "times;" they should learn to read and use it in either way, as occasion require.

C. 1. * × 4 = 16?
2. * × 3 = 18?
3. * × 4 = 36?
4. * × 5 = 20?
5. 4 × * = 40?
6. 5 × * = 20?
7. 4 × * = 28?
8. 2 × * = 12?
9. 40 = * × 4?
10. 28 = * × 4?
11. 24 = * × 4?
12. 32 = * × 8?
13. 32 = 4 × *?
14. 36 = 4 × *?
15. 24 = 6 × *?
16. 16 = 4 × *?

D. 1. 7 times 4, plus 8 = * times 9?
2. 9 times 3, plus 5 = * times 4?
3. 5 times 3, plus 3 times 7 = * times 4?
4. 2 plus 7, plus 6 times 3 = * times 8?

E. 1. 3 pk. 6 qt. = * times 5 qt.?

Solution.—3 pk. 6 qt. = 30 qt.; and 30 qt. contains 5 qt. as m times as 30 contains 5, which are 6 times. Hence 3 pk. 6 qt. = 6 ti 5 qts.

Abbreviated Solution.—3 pk. 6 qt. = 30 qt.; and 30 qt. = 6 ti 5 qt. Hence 3 pk. 6 qt. = 6 times 5 qt.

2. 3 wk. 3 da. = * times 4 da.?
3. 7 gal. 2 qt. = * times 3 qt.?
4. 2 wk. 4 da. = * times 6 da.?
5. 6 ℥ 2 ℈ = times 2 ℈?
6. 8 yd. = * times 1 yd. 1 ft.?
7. 4 gal. 2 qt. = * times 1 gal. 2 qt.

8. Edward can walk 4 miles per hour, and Herman can walk 3. How far can Edward walk in 6 hours? Can Herman?

9. Richard bought 8 newspapers at 2 cents a-piece, and sold them for 4 cents a-piece. How much did he gain on them?

10. If Daniel has 50 chestnuts, how many will he have left after giving 4 of his companions 9 chestnuts a-piece?

11. I bought 9 yards of cloth at $4 per yard, but it being damaged, I was obliged to sell it for $12 less than it cost me. For how much did I sell it?

12. 1 bushel = * quarts?
13. 1 yard = * nails?
14. 4 pt. = * gills?

15. Arthur had a basket which held just 4 qt., and he picked nuts enough to fill it 6 times. How many quarts did he pick? How many pecks?

16. How many oranges at 4 cents each can be bought for 6 three-cent pieces and 2 cents?

17. How many apples at 3 cents a-piece can be bought for 6 oranges at 4 cents each?

18. How many pairs of boots at $5 a pair, can be bought for 10 yards of cloth at $3 per yard?

19. A man bought 10 quarts of berries, which he put into boxes each holding 5 pints. How many pints of berries did he buy? How many boxes did he fill?

20. Lucius is shelling corn into a three-peck measure, which he empties into a bin large enough to hold 6 bushels 3 pecks. How many pecks must he shell to fill the bin? How many measurefuls?

21. A newsboy sold 9 papers at 3 cents a-piece, and after spending 9 cents, gave the rest of his money for papers at 2 cents a-piece. How many papers did he get?

22. A man gave 8 hats at $4 a-piece, and $8 in money for at $10 a-piece. How many coats did he receive?

23. A man put 7 gallons 2 quarts of molasses into jugs holding 3 quarts. How many jugs did he fill?

24. Rufus had a string 5 yards 1 foot long, which he cut pieces just 2 feet long. How many pieces did it make?

25. An apothecary put 5 ʒ 1 ℈ of powders into papers, holding 2 ℈. How many papers did he fill?

LESSON XXXIII.

A. Add 10 fives together.

5 times 5 = 25.
5 times 6, or 6 times 5 = 30.
5 times 7, or 7 times 5 = 35.
5 times 8, or 8 times 5 = 40.
5 times 9, or 9 times 5 = 45.
5 times 10, or 10 times 5 = 50.

B. Add 10 sixes together.

6 times 6 = 36.
6 times 7, or 7 times 6 = 42.
6 times 8, or 8 times 6 = 48.
6 times 9, or 9 times 6 = 54.
6 times 10, or 10 times 6 = 60.

C. 1. * × 6 = 36?
2. * × 8 = 48?
3. * × 5 = 45?
4. * × 6 = 30?
5. 42 = * × 7?
6. 54 = * × 9?
7. 54 = * × 6?
8. 25 = * × 5?

D. 1. 9 × * = 54?

2. 7 × * = 35?

3. 6 × * = 48?

4. 8 × * = 40?

5. 60 = 6 × *?

6. 40 = 8 × *?

7. 36 = 6 × *?

8. 42 = 7 × *?

EXPLANATION.—To divide a number by 2 is to find how many times 2 equal it.

To divide a number by 6 is to find how many times 6 equal it.

Hence, 35 divided by 5 = 7, for 35 = 7 times 5; 48 divided by 8 = 6, for 48 = 6 times 8.

Division, then, is the process of finding how many times one number must be taken to equal another number

The number to be divided is called the DIVIDEND. The number by which we divide is called the DIVISOR, and the answer is called the QUOTIENT.

Thus, in 35 divided by 7 = 5, 35 is the dividend, 7 is the divisor, 5 is the quotient.

In 54 = * times 6, 54 is the dividend, 6 is the divisor, and 9, the answer, is the quotient.

Name the divisor, dividend, and quotient of the following examples:—

28 divided by 7 = 4.

48 divided by 6 = 8.

35 divided by 7 = 5.

36 = * × 6?

20 = * × 5?

28 = * × 7?

By these illustrations it appears that division is just the reverse of multiplication.

A horizontal line with one dot above, and another below it, forms the sign of division, thus: ÷. It may be read "divided by."

28 ÷ 7 = 4 may be read, "28 divided by 7 equal 4."

F. What is the quotient of —

1. 16 ÷ 4?
2. 24 ÷ 3?
3. 32 ÷ 8?
4. 25 ÷ 5?
5. 48 ÷ 6?
6. 42 ÷ 7?
7. 18 ÷ 3?
8. 24 ÷ 8?
9. 54 ÷ 9?

G. 1. 5 wk. 1d. ÷ 1 wk. 2 da.?

Solution. — 5 wk. 1 da. — 36 days; 1 wk. 2 da. = 9 days; da. = 4 times 9 da. Hence, 5 wk. 1 da. ÷ 1 wk. 2 d. = 4.

2. 3 pk. 6 qt. ÷ 1 pk. 2 qt.

3. 6 yd. 2 ft. ÷ 1 yd. 2 ft.

4. 9 gal. ÷ 2 gal. 1 qt.

5. 6 wk. 3 da. ÷ 1 wk. 2 da.

6. 8 ℥ 6 ʒ ÷ 1 ℥ 2 ʒ.

7. 3 sq. yd. 5 sq. ft. ÷ 8 sq. ft.

8. I sold 7 quarts of cherries at 6 cents per quart, an quart for 8 cents. How much did I receive? I expend money thus received for rice at 5 cents per pound. How pounds of rice did I buy?

9. If George walks at the rate of 15 rods per minute, an liam walks at the rate of 21 rods per minute, how many mo per minute does William walk than George? How many rods in 9 minutes.

10. If Susan gains 8 merit-marks per day, and loses 2 pe how many will she have at the end of 8 days?

11. A man sold 5 pecks of chestnuts at the rate of on per quart. How many dimes did he receive?

12. I sold 6 quarts of blackberries at the rate of 10 cen quart, and received in payment 5 three-cent pieces, and th in half-dimes. How many half-dimes did I receive?

13. Abner and Lemuel were in a store together, and their told them that they might each have 8 oranges worth 6 ce piece, or 9 worth 5 cents a-piece. Abner chose the forme Lemuel the latter. How much more were Abner's oranges than Lemuel's?

14. How many bags, each containing 1 bu. 1 pk., can b from 8 bu. 8 pk. of meal?

15. How many house-lots, each containing 1 A. 2 R., can be made from a piece of land containing 10 A. 2 R.?

16. How many pictures at 2d. 1 qr. each can be purchased for 6d. 3 qr.?

17. How many bushels in 8 bags, each containing 3 pecks?

18. A furniture-dealer gave 6 bureaus, worth 7 dollars a-piece, and 3 dollars in money, for chairs at 9 dollars per dozen. How many dozen chairs did he buy?

19. A fur-dealer gave 8 caps, at 5 dollars a-piece, and 2 dollars in money, for muffs at 6 dollars a piece. How many muffs did he receive?

20. A boy earned 12 cents by doing some errands, and invested the money in papers at 2 cents a-piece. He sold the papers at 4 cents each, and with the money received for them he bought papers at 3 cents a-piece. He sold 6 of the papers for 5 cents a-piece, and the rest for 2 cents a-piece. He then spent 4 cents for crackers, and gave the rest of his money for some very nice Havana oranges at 6 cents a-piece. He gave 1 of the oranges to his mother, and sold the rest at 8 cents a-piece. How much did he receive for them?

LESSON XXXIV.

A. 7 times 7 = 49.
7 times 8, or 8 times 7 = 56.
7 times 9, or 9 times 7 = 63.
7 times 10, or 10 times 7 = 70.

8 times 8 = 64.
8 times 9, or 9 times 8 = 72.
8 times 10, or 10 times 8 = 80.

9 times 9 = 81.

9 times 10, or 10 times 9 = 90.

10 times 10 = 100.

B. 56 = * × 8?

81 = * × 9?

63 = * × 7?

64 = * × 8?

90 = * × 10?

49 = * × 7?

100 = * × 10?

72 = * × 8?

C. 81 ÷ 9?

36 ÷ 4?

48 ÷ 8?

72 ÷ 9?

56 ÷ 8?

21 ÷ 7?

45 ÷ 5?

81 ÷ 9?

100 ÷ 10?

24 ÷ 8?

72 ÷ 9?

63 ÷ 7?

D. 1. 7 times 8, plus 7, divided by 7, multiplied by 4 = times 6?

Solution.—7 times 8 = 56, plus 7 = 63, divided by 7 = 9, mu plied by 4 = 36, which equals 6 times 6.

Note.—Let the pupil learn to call only the *results*, thus: 56, 9, 36, 6. Let them also perform the work as the Teacher reads example.

2. 8 times 10, minus 8, divided by 9, plus 1, multiplied by minus 5 = * times 7?

3. 6 times 8, minus 12, divided by 4, multiplied by 3, plus times 4, minus 3 = * times 8?

4. 7 times 6, plus 3, divided by 9, multiplied by 5, plus divided by 7, multiplied by 5, minus 2 = * times 2?

5. 3 times 8, plus 4 times 9, plus 21, divided by 9, multipli by 2, plus 7, divided by 5?

E. 1. How many are 9 times 2 yds. 2 ft.?

Solution. — 9 times 2 yd. = 18 yd.; 9 times 2 ft. = 18 ft. = yds., which, added to 18 yd. = 24 yd. Hence, 9 times 2 yd. 2 = 24 yd.

2. 4 times 7 gal. 3 qt.?
3. 7 times 3 wk. 4 da.?
4. 6 times 9 yd. 2 ft.?
5. 9 times 8 sq. yd. 4 sq. ft.?
6. 6 times 7 bu. 6 pk.?
7. 6 times 8 gal. 2 qt.?

8. Bought 6 bags, each containing 3 bu. 2 pk. of peanuts, and put them into casks each holding 3 bushels. How many casks did they fill?

9. Austin had a basket which held just 3 pk. 4 qt., and another which held just 5 pecks. He gathered nuts enough to fill the smallest basket 10 times. How many times would they fill the large basket?

10. A lady bought cotton sheeting enough to make 8 sheets, each containing 5 yd. 1 qr., but afterwards concluded to put 6 yards in a sheet. How many sheets could she make?

11. I bought a vessel of oil containing 32 quarts, and after using 1 gal. 3 qts. of it, I sold the rest at the rate of 1 dollar for 1 gal. 1 qt. How much did I get for it?

12. Sarah's mother offered to give her 8 large oranges worth 5 cents a-piece if she would tell her how many seven-quart baskets could be filled from 5 pk. 2 qt. of berries. Sarah answered correctly. What was her answer? She exchanged the oranges for their value in drawing-pencils worth 10 cents a-piece. How many pencils did she get?

13. A fruit-peddler paid 48 cents for cherries at 6 cents a quart, which he put into papers, each containing 1 gill. How many papers did it take?

14. By buying a lot of wood at $4 per cord, and selling it at $6 per cord, I gained $18. How many dollars did I gain on each cord? How many cords did I buy? How many dollars did I pay for the entire lot?

15. By buying flour at $5 per barrel, and selling it for $8 per

barrel, I gained $24. How many barrels did I buy, and w I pay for the lot?

16. By buying a lot of cloth for $5 per yard, and sellii $8 per yard, I lost $20. How many dollars did I pay lot?

17. A man bought a lot of coal at $4 per ton, and se $8 per ton, by which he gained $36. How many dollars pay for it?

18. A laborer worked 6 weeks for 9 dollars per week, a ting 6 dollars with the money thus earned, he bought c dollars per ton. How many tons did he buy? After layi 2 tons for his own use, he sold the remainder for 7 doll ton, receiving in payment 2 dollars in money, and the flour at 9 dollars per barrel. How many barrels of flour receive?

19. Robert had a basket holding 2 quarts 1 pint, gathered chestnuts enough to fill it 8 times. How ma quart baskets could he fill with what he gathered?

20. Augustus had 8 quarts of blackberries. He sold for 11 cents, and the rest for 10 cents per quart. With th received for them he bought cocoa-nuts at 9 cents a-piece many cocoa-nuts did he buy? He divided 2 of them am companions, and sold the rest for 10 cents a-piece. He cents to a poor woman, and spent the rest of his money crackers, at 7 cents a bunch. How many bunches did he

LESSON XXXV.

Bills.

A. 1. Mr. Edward Crane keeps a store in Boston. On the 21st of February, 1856, he sold to Mr. Alfred Hall 7 yards of broadcloth at $5 per yard, 9 yards of cassimere at $2 per yard, and 8 yards of doeskin at $3 per yard. Mr. Hall paid for them, and asked Mr. Crane to make out a *bill*, *i. e.*, a written statement of the transaction. He did it as follows :—

Boston, Feb. 21, 1856.

Mr. Alfred Hall

Bought of Edward Crane.

7 yds. Broadcloth, a $5 . . .	*$35*
9 yds. Cassimere, a $2 . .	*18*
8 yds. Doeskin, a $3 . . .	*24*
	$77

Received Payment,

Edward Crane.

If the goods had not been paid for, Mr. Crane would not have *receipted* the bill, *i. e.*, he would not have put his name after the words "received payment."

NOTE TO THE TEACHER. — A more full explanation may be found in "ARITHMETIC AND ITS APPLICATIONS."

B. Make out proper bills for each of the following transac observing to give: —

1st. The date, *i. e.*, the place and time.

2d. The names of the parties.

3d. The articles bought, with their prices and amount.

4th. The words "*received payment.*"

5th. *If the goods are paid for, or bought for cash*, the name seller.

NOTE.—It may add to the interest and value of the followin blems, to have each pupil write his own name, and that of some panion, in the place of those here written.

1. John Brown, of New York, sold to Martin Draper, for July 16th, 1856, 9 bbls. of flour at $10 per barrel, and 8 b of wheat at $2 per bushel.

2. William Fuller, of Chicago, sold to Arthur Simmons for Jan. 8th, 1856, 9 silk hats at $4 a-piece, 8 cloth caps a-piece, 6 beaver hats at $5 a-piece, and 11 drab hats a-piece.

3. Henry Mitchel & Co. sold to Francis Baker, ON CREDIT Mr. Baker at the time did not pay for them), May 1st, 1 bbls. of apples at $3 per barrel, 9 bbls. of potatoes at $ barrel, 7 boxes of raisins at $3 per box, and 5 drums of $2 per drum.

NOTE. — Let the students now make out bills for several ima transactions.

C. If in the transaction described under letter A., Mr. Ha paid $2 in money and delivered to Mr. Crane 8 cords of wo $9 per cord, the bill would have been made out as follows:

Boston, Feb. 21, 1856.

Mr. Alfred Hall

Bought of Edward Crane.

7 yds. Broadcloth, a $5	$35
9 yds. Cassimere, a $2	18
8 yds. Doeskin, a $3	24
	$77

Cr.

By Cash	$12
By 3 cords wood, a $8,	24
	$36
Balance due E. C.	$41

Make out a bill for the following transaction:—

1. Moses White, a trader of Philadelphia, sold to Joseph Austin, May 17th, 1856, 7 ploughs at $9 each, 5 hay-cutters at $8 a-piece, 4 doz. scythes at $9 per dozen, and 8 doz. rakes at $3 per dozen; and in part payment, Mr. Austin gave him 8 cords of wood at $7 per cord, 4 cords of wood at $6 per cord, and $18 in money.

NOTE.—Let the scholars make out bills for several imaginary transactions.

LESSON XXXVI.

A. 1. 29 = * times 6 ?

First Solution.—29 = 24 + 5, and 24 = 4 times 6. Hence 2 = 4 times 6 with 5 remainder.

Second Solution.—29 = 4 times 6 with 5 remaining, for 4 time 6 = 24, and 5 = 29.

Note.—The remainder is really an undivided part of the dividend and might be subtracted from it without affecting the quotient. I reality, but a part of the dividend is divided.

2. 46 = * times 5 ?
3. 31 = * times 9 ?
4. 57 = * times 7 ?
5. 88 = * times 9 ?
6. 61 = * times 8 ?
7. 37 = * times 7 ?
8. 53 = * times 6 ?
9. 27 = * times 8 ?
10. 52 = * times 9 ?
11. 48 = * times 7 ?
12. 69 = * times 9 ?
13. 23 = * times 11 ?

B. 1. 42 ÷ 4 ?
2. 83 ÷ 9 ?
3. 75 ÷ 8 ?
4. 24 ÷ 7 ?
5. 19 ÷ 8 ?
6. 47 ÷ 7 ?
7. 39 ÷ 6 ?
8. 51 ÷ 8 ?
9. 43 ÷ 9 ?
10. 43 ÷ 4 ?
11. 47 ÷ 8 ?
12. 47 ÷ 6 ?

C. The comma, when used in connexion with Arithmetical sign shows that the *result* of all the preceding operations is to be considered in connexion with the sign following it.

Thus: "4 × 9, ÷ 7," means that 4 times 9, or 36, is to be divide by 7.

"6 × 7 + 18, ÷ 9," means that the sum of 6 times 7 plus 18, = 42 + 18 = 60, is to be divided by 9.

1. $5 \times 9, \div 8$?
2. $4 \times 6, \div 7$?
3. $5 \times 9 + 8, \div 6$?
4. $8 \times 8 - 13, \div 9$?
5. $9 \times 6 + 10, = * \times 8$?
6. $4 \times 9 + 17, = * \times 6$?
7. $7 \times 7 + 13, = * \times 10$?
8. $9 \times 9 - 43, = * \times 4$?

D. 1. 41 da. = * wk.?

Solution.—Since 7 days = 1 wk., 41 da. must equal as many wk. as there are times 7 in 41, which are 5 times with 6 remainder. Hence 41 da. = 5 wk. 6 da.

2. 33 qr. = * yd.?
3. 37 ʒ = * ℥?
4. 70 qt. = * pk.?
5. 19 ft. = * yd.?
6. 28 ft. = * yd.?
7. 46 fur. = * m.?
8. 59 da. = * wk.?
9. 37 cd. ft. = * cd.?
10. 60 sq. ft. = * sq. yd.?
11. 20 m. = * le.?

12. Moses has 35 cents, with which he wishes to buy oranges at 6 cents a-piece. How many oranges can he buy?

Solution.—If he can buy 1 orange for 6 cents, he can buy as many oranges for 35 cents as there are times 6 in 35, which are 5 times and 5 remainder. Therefore, he can buy 5 oranges, and have 5 cents remaining.

13. How many barrels of flour, at $6 per barrel, can be bought for $58?

14. How many shawls, at $7 a-piece, can be bought for $39?

15. How many books, at $8 a-piece, can be bought for $29?

16. How many terms tuition, at $7 per term, will $27 pay for?

17. How many hats, at $4 each, can be bought for $39?

18. How many cans, each holding 1 gal. 1 qt., can be filled from 3 gal. 3 qt. of milk?

19. A person who owes $39, wishes to pay as much as possible in five-dollar bills, and the rest in one-dollar bills. How many bills of each kind must he pay?

20. William did 8 errands, for each of which he received 3 cents, and he had 11 cents before he did the errands. He bought as many writing-books at 9 cents a-piece as he could pay for, and spent the rest of his money for pens at 2 cents a-piece. How many writing-books did he buy? How many pens?

21. A tailor paid $35 for silk velvet at $5 per yard. He made it into vests, putting 3 quarters into each vest. How many vests did he make, and how many quarters had he remaining?

22. One "Fourth of July" Thomas had 29 cents. He bought as many bunches of crackers at 10 cents per bunch, as he could pay for, and then spent the rest of his money for cherries, at the rate of 7 for a cent. How many bunches of crackers did he buy? How many cherries?

23. A person who had 20 cents, said to a boy: "If you will tell me how many loaves of bread, at 6 cents per loaf, I can buy with my money, I will give you what there is left after paying for the bread." The boy answered right. What was his answer? How many cents ought the person to give him?

24. If it requires 3 yards of broadcloth to make a coat, how many coats can be made from a piece containing 29 yards of broadcloth? How many yards will be left after making the coats? If one yard of cloth will make 3 vests, how many vests can be made from what remains, after making the coats?

25. Lyman has 28 cents, Horace has 50. Chester has 63, and Isaac has 47. Each bought as many pencils at 8 cents a-piece as he could pay for, and gave the rest of his money to a poor woman. How many pencils did each buy, and how many cents had each to give the poor woman? How many pencils were bought in all? How many cents did the woman receive?

LESSON XXXVII.

A. 1. 2 times 3 tens? Then 3 times 30?
2. 5 times 7 tens? Then 5 times 70?
3. 3 times 4 tens? Then 3 times 40?
4. 7 times 3 tens? Then 7 times 30?

B. 1. 4 × 3?
2. 4 × 30?
3. 40 × 3?
4. 7 × 9?
5. 7 × 90?
6. 70 × 9?
7. 8 × 7?
8. 8 × 70?
9. 80 × 7?

C. 1. 5 × 40?
2. 9 × 30?
3. 6 × 40?
4. 40 × 4?
5. 6 × 30?
6. 60 × 3?
7. 9 × 70?
8. 90 × 7?
9. 9 × 60?
10. 8 × 90?
11. 7 × 70?
12. 8 × 80?

D. 1. 12 ÷ 4?
2. 120 ÷ 4?
3. 120 ÷ 40?
4. 64 ÷ 8?
5. 640 ÷ 8?
6. 640 ÷ 80?
7. 54 ÷ 9?
8. 540 ÷ 90?
9. 540 ÷ 9?

E. 1. 630 ÷ 9?
2. 720 ÷ 8?
3. 560 ÷ 7?
4. 250 ÷ 5?
5. 240 ÷ 3?
6. 720 ÷ 80?
7. 180 ÷ 3?
8. 420 ÷ 60?
9. 90 ÷ 30?
10. 270 ÷ 30?
11. 420 ÷ 6?
12. 810 ÷ 9?
13. 810 ÷ 90?
14. 360 ÷ 40?

F. 1. 6 × 47?

SOLUTION.—6 times 40 = 240; 6 times 7 = 42, which, added to 240 = 282. Therefore, 6 times 47 = 282.

Note. — As soon as the principle is understood, the pupil should solve such problems by naming only the results. Thus: 6 times 47 = 240 + 42 = 282.

2. 7 × 96?
3. 8 × 34?
4. 9 × 37?
5. 6 × 94?
6. 8 × 23?
7. 4 × 27?
8. 9 × 82?
9. 5 × 97?
10. 4 × 23?
11. 7 × 94?
12. 8 × 37?
13. 6 × 28?
14. 5 × 43?
15. 9 × 81?
16. 7 × 63?

G. When we wish to write the work, we may, if we choose, solve examples in multiplication, as explained in the following solution of the first question under F.

47
6
———
282

Explanation.—6 times 7 units = 42 units = 4 tens, and 2 units. Writing the 2 units, and reserving the 4 tens to add to the product of the tens, we have 6 times 4 tens = 24 tens, to which, adding the 4 tens from the former product, gives 28 tens, which we write. The answer, then, is 282.

Note. — It will be seen that when we *do not* write the work, we begin at the *left* hand to multiply; and when we *do* write it, we begin at the right hand.

Perform in this way the examples under letter F.

H. 1. 6 times 498?

Solution. — 6 times 8 units = 48 units = 4 tens and 8 units. Writing the 8 units, and reserving the 4 tens to add to the product of the tens, we have 6 times 9 tens = 54 tens, and 4 tens added, equal 58 tens = 5 hundreds and 8 tens. Writing 8 tens, and reserving the 5 hundreds to add to the product of the hundreds' column, we have 6 times 4 hundreds = 24 hundreds, and 5 hundreds added = 29 hundreds = 2 thousands and 9 hundreds, which being the last product we write, the answer then is 2988.

498
6
———
2988

NOTE.—When the reductions are fully mastered, abbreviated forms ke the following may be introduced with advantage.
6 times 8 = 48. Write 8, and add 4 to the next product. 6 times are 54 and 4 are 58. Write 8 and add 5 to the next product. 6 times are 24 and 5 are 29, which we write. Hence, 6 times 498 = 2988.

The following form of naming only results should finally be dopted. 48 units; 54, 58 tens; 24, 29 hundreds. *Ans.*—2988.

2.	9 × 847?	8.	6 times $2.75?
3.	3 × 298?	9.	4 times $8.76?
4.	4 × 746?	10.	9 times $32.75?
5.	8 × 327?	11.	8 times $27.84?
6.	4 × 238?	12.	5 times $97.83?
7.	5 × 379?	13.	4 times $28.59?

LESSON XXXVIII.

A. 1. 476 ÷ 7?

SOLUTION.—7 is contained in 47 tens, 6 tens' times, with 5 tens remaining. 5 tens = 50 units, and 6 units added, are 56 units, 7 is ontained in 56 units 8 units' times. Hence, 476 ÷ 7 = 60 + 8 = 68.

NOTE.—Not till the reductions are fully understood, should the upil be allowed to abbreviate this explanation to the common one: "7 contained in 47, 6 times, with 5 remainder. 7 is contained in 56, 8 imes. Hence the quotient is 68."

2.	315 ÷ 3?	6.	672 ÷ 8?	10.	429 ÷ 8?
3.	216 ÷ 4?	7.	144 ÷ 6?	11.	418 ÷ 5?
4.	392 ÷ 8?	8.	279 ÷ 9?	12.	678 ÷ 7?
5.	217 ÷ 7?	9.	137 ÷ 2?	13.	528 ÷ 9?

Examples in division are performed and explained in the sa
manner when we write the work as when we do not. The wo
of the first example, letter A., would usually be written as in t
annexed model:

$$\begin{array}{r} 7)\underline{476} \\ 68 \end{array}$$

B. 1. 2738 ÷ 8?

$$\begin{array}{l} 8)\underline{2738}—2 \\ \quad 342 \end{array}$$

SOLUTION.—8 is contained in 27 hundreds, 3 hundred times with
hundreds remaining. We therefore write 3 as the hundreds' fig
of the quotient. The 3 hundreds
maining = 30 tens, and 3 tens add
= 33 tens. 8 is contained in 33 te
4 tens times, with 1 ten remaini
We therefore write 4 as the tens fig
of the quotient. The 1 ten remaining = 10 units, and 8 units added
18 units. 8 is contained in 18 units 2 units' times and 2 units
maining. Therefore, 2738 ÷ 8 = 3 hundreds, 4 tens, and 2 units,
342 with a remainder of 2.

NOTE.—The remainder is written after the quotient with the si
of subtraction, to show that it is an undivided part of the dividend.

2. 4756 ÷ 4?
3. 3297 ÷ 6?
4. 4347 ÷ 9?
5. 2981 ÷ 11?
6. 3297 ÷ 6?
7. 4361 ÷ 5?
8. 2459 ÷ 8?
9. 4272 ÷ 12?
10. 8943 ÷ 9?
11. 2137 ÷ 5?
12. 4264 ÷ 13?
13. 8375 ÷ 12?
14. 2986 ÷ 4?
15. 3176 ÷ 8?
16. 4327 ÷ 9?
17. 2052 ÷ 8?
18. 1379 ÷ 2?
19. 7436 ÷ 8?

LESSON XXXIX.

The following tables, if thoroughly learned, will save a vast deal of labor in the Arithmetical operations of life. A distinguished educator, now Superintendent of Schools in one of the principal cities of the Union, says that in his opinion, a knowledge of these tables would save hours of valuable time, not only to the student, but to the business man. With most classes, the Teacher will find it desirable to give additional exercises similar to those of the preceding Lessons.

11 × 2, or 2 × 11 = 22.
12 × 2, or 2 × 12 = 24.
13 × 2, or 2 × 13 = 26.
14 × 2, or 2 × 14 = 28.
15 × 2, or 2 × 15 = 30.

16 × 2, or 2 × 16 = 32.
17 × 2, or 2 × 17 = 34.
18 × 2, or 2 × 18 = 36.
19 × 2, or 2 × 19 = 38.
20 × 2, or 2 × 20 = 40.

11 × 3, or 3 × 11 = 33.
12 × 3, or 3 × 12 = 36.
13 × 3, or 3 × 13 = 39.
14 × 3, or 3 × 14 = 42.
15 × 3, or 3 × 15 = 45.

16 × 3, or 3 × 16 = 48.
17 × 3, or 3 × 17 = 51.
18 × 3, or 3 × 18 = 54.
19 × 3, or 3 × 19 = 57.
20 × 3, or 3 × 20 = 60.

11 × 4, or 4 × 11 = 44.
12 × 4, or 4 × 12 = 48.
13 × 4, or 4 × 13 = 52.
14 × 4, or 4 × 14 = 56.
15 × 4, or 4 × 15 = 60.

16 × 4, or 4 × 16 = 64.
17 × 4, or 4 × 17 = 68.
18 × 4, or 4 × 18 = 72.
19 × 4, or 4 × 19 = 76.
20 × 4, or 4 × 20 = 80.

11 × 5, or 5 × 11 = 55.
12 × 5, or 5 × 12 = 60.
13 × 5, or 5 × 13 = 65.
14 × 5, or 5 × 14 = 70.
15 × 5, or 5 × 15 = 75.

11 × 6, or 6 × 11 = 66.
12 × 6, or 6 × 12 = 72.
13 × 6, or 6 × 13 = 78.
14 × 6, or 6 × 14 = 84.
15 × 6, or 6 × 15 = 90.

11 × 7, or 7 × 11 = 77.
12 × 7, or 7 × 12 = 84.
13 × 7, or 7 × 13 = 91.
14 × 7, or 7 × 14 = 98.
15 × 7, or 7 × 15 = 105.

11 × 8, or 8 × 11 = 88.
12 × 8, or 8 × 12 = 96.
13 × 8, or 8 × 13 = 104.
14 × 8, or 8 × 14 = 112.
15 × 8, or 8 × 15 = 120.

11 × 9, or 9 × 11 = 99.
12 × 9, or 9 × 12 = 108.
13 × 9, or 9 × 13 = 117.
14 × 9, or 9 × 14 = 126.
15 × 9, or 9 × 15 = 135.

16 × 5, or 5 × 16 = 8
17 × 5, or 5 × 17 = 8
18 × 5, or 5 × 18 = 9
19 × 5, or 5 × 19 = 9
20 × 5, or 5 × 20 = 1

16 × 6, or 6 × 16 = 9
17 × 6, or 6 × 17 = 1
18 × 6, or 6 × 18 = 1
19 × 6, or 6 × 19 = 1
20 × 6, or 6 × 20 = 1

16 × 7, or 7 × 16 = 1
17 × 7, or 7 × 17 = 1
18 × 7, or 7 × 18 = 1
19 × 7, or 7 × 19 = 1
20 × 7, or 7 × 20 = 1

16 × 8, or 8 × 16 = 1
17 × 8, or 8 × 17 = 1
18 × 8, or 8 × 18 = 1
19 × 8, or 8 × 19 = 1
20 × 8, or 8 × 20 = 1

16 × 9, or 9 × 16 = 1
17 × 9, or 9 × 17 = 1
18 × 9, or 9 × 18 = 1
19 × 9, or 9 × 19 = 1
20 × 9, or 9 × 20 = 1

11×10, or $10 \times 11 = 110$.

12×10, or $10 \times 12 = 120$.

13×10, or $10 \times 13 = 130$.

14×10, or $10 \times 14 = 140$.

15×10, or $10 \times 15 = 150$.

16×10, or $10 \times 16 = 160$.

17×10, or $10 \times 17 = 170$.

18×10, or $10 \times 18 = 180$.

19×10, or $10 \times 19 = 190$.

20×10, or $10 \times 20 = 200$.

LESSON XL.

Note.—Should the Teacher deem it best, the class may omit this and the next Lesson, till after some of the first Lessons on Fractions have been learned.

A. From the exercises of Lesson XXXVII., B. and C., we may infer that 4 times 30 = 40 times 3; that 70 times 9 = 7 times 90, &c., &c. In like manner, 40 times 27 = 4 times 270, or, 4 tens' times 27 = 4 times 27 tens; 80 times 436 = 8 times 4360, or, 8 tens' times 436 = 8 times 436 tens, &c., &c.

1. What is the product of 70 times 389?

Solution. — 70 times 389 = 7 times 3890, which may be found by the method explained in Lesson XXXVII., G. and H. Thus: 7 times 0 units = 0 units. 7 times 9 tens = 63 tens = 6 hundreds and 3 tens, &c., &c.

$$\begin{array}{r} 389 \\ 70 \\ \hline 26930 \end{array}$$

2. 20 times 64?
3. 80 times 29?
4. 40 times 36?
5. 60 times 94?
6. 90 times 37?
7. 20 times 93?
8. 30 times 84?
9. 90 times 72?
10. 30 times 979?
11. 40 times 832?
12. 70 times 697?
13. 20 times 443?
14. 60 times 927?
15. 80 times 423?
16. 50 times 975?
17. 80 times 476?

B. Since 24 = 20 + 4, 24 times any number must equal 2 times that number plus 4 times that number. Since 86 = 80 + 6, 86 times any number must equal 80 times that number plus times that number, &c., &c.

1. What is the product of 29 times 863 ?

$$\begin{array}{r} 863 \\ 29 \\ \hline 7767 \\ 1726 \\ \hline 25027 \end{array}$$

Solution. — Since 29 = 20 + 9, 29 time 863 must equal 20 times 863, plus 9 times 863 We first multiply by 9, and then by 20, by th methods before explained, and add the product together as in the written work at the left.

2. 38 times 481 ?
3. 27 times 936 ?
4. 68 times 427 ?
5. 43 times 268 ?
6. 31 times 492 ?
7. 68 times 946 ?
8. 79 times 368 ?
9. 42 times 427 ?
10. 54 times 329 ?
11. 61 times 428 ?
12. 89 × 2796 ?
13. 38 × 9582 ?
14. 22 × 4858 ?
15. 56 × 9375 ?
16. 46 × 2401 ?
17. 63 × 2485 ?
18. 81 × 3258 ?
19. 69 × 2846 ?
20. 44 × 8132 ?
21. 74 × 9128 ?

LESSON XLI.

A. When the divisor is a large number, it is often convenien or necessary to use the nearest number of tens, hundreds, or thou sands, as a *trial divisor*, to determine the *probable* quotient figur

Illustrations.—In dividing by 31, 32, 33, or 34, we may make 30 or 3 the trial divisor. In dividing by 36, 37, 38, or 39, we may make 40 or 4 the trial divisor. In dividing by 35, we may make either 30 or 40 the trial divisor.

B. 1. What is the quotient of 178 ÷ 53?

Solution.—We may make 50 or 5 the trial divisor, for 53 is contained in 178 about the same number of times that 50 is; or, that 5 is contained in 17, which is 3 times. To test the correctness of this conclusion, we must find 3 times 53. It is 159, which, subtracted from 178, leaves 19, thus showing that 178 ÷ 53 = 3 with 19 remainder.

The work would be written by placing the divisor at the left of the dividend, the quotient at the right, and the product with the remainder beneath the product.

53)178(3
159 = 3 × 53
19—Rem.

Note. — The Teacher should illustrate and explain the method of proceeding when the above process gives a trial quotient figure either too large or too small. [See "Arithmetic and its Applications," 91st article, and solution to 2d example, 113th page, and to 4th example, 114th page.]

2. 96 ÷ 24?
3. 127 ÷ 31?
4. 228 ÷ 64?
5. 588 ÷ 82?
6. 281 ÷ 29?
7. 469 ÷ 48?
8. 356 ÷ 61?
9. 429 ÷ 57?
10. 256 ÷ 38?
11. 124 ÷ 19?
12. 337 ÷ 45?
13. 621 ÷ 84?
14. 438 ÷ 62?
15. 279 ÷ 94?
16. 349 ÷ 82?
17. 624 ÷ 79?

C. What is the quotient of 2856 ÷ 59?

59)2856(48
236
496
472
24 = Rem.

Explanation. — 59 is s near 60, that we make 6 th trial divisor. Since 6 is con tained 4 times in 28, we mak 4 the first figure of the quc tient, and infer that 59 is cor tained 4 tens' times in 28 tens. Multiplying 59 by tens, gives 236 tens. Henc we write 236 under the 28 and subtract the former from the latter. It leaves a remainder of 4 or 49 tens, 490 units, to which, adding the 6 units, gives 496 unit Since 6 is contained 8 times in 49, we make 8 the next figure of tl quotient, and infer that 59 is contained 8 times in 496. Multiplyi 59 by 8, gives 472; hence we write 472 under the 496, and subtra the former from the latter. It leaves a remainder of 24. Henc 2856 ÷ 59 = 48 with 24 remainder.

Proof.—48 times 59, plus 24, equals 2856.

2. 848 ÷ 31?
3. 579 ÷ 43?
4. 827 ÷ 15?
5. 1748 ÷ 42?
6. 3947 ÷ 49?
7. 8246 ÷ 91?
8. 4217 ÷ 88?
9. 8321 ÷ 94?
10. 6785 ÷ 88?
11. 2817 ÷ 88?
12. 7635 ÷ 82?
13. 1749 ÷ 22?
14. 2175 ÷ 25?
15. 4802 ÷ 49?
16. 6237 ÷ 74?
17. 4238 ÷ 52?
18. 5947 ÷ 75?
19. 3286 ÷ 47?

LESSON XLII.

A. 1. The multiplier and multiplicand are called FACTORS of their product. (See Lesson XXXII., B.)

2. The FACTORS of any number are the numbers which, multiplied together, will give that number for a product.

ILLUSTRATIONS. — 5 and 3 are factors of 15, because $15 = 5 \times 3$.
Again, 6 and 2, 3 and 4, or 3, 2, and 2, are factors of 12, because $12 = 6 \times 2, = 3 \times 4 = 3 \times 2 \times 2$.
Again, 2 is a factor of 4, 6, 8, 10, &c. 3 is a factor of 6, 12, 15, 18, &c.

3. From the above illustrations, we see that the *factors* of a number are DIVISORS of it, *i. e.*, they are such numbers as will divide it without a remainder.

4. A prime number is a number which has no factors except itself and 1.

ILLUSTRATIONS.—1, 2, 3, 5, 7, and 11, are each prime numbers.

5. A composite number is a number which has other factors besides itself and 1.

ILLUSTRATIONS. — 4, 6, 8, and 9, are each composite numbers, for $4 = 2 \times 2$, $6 = 3 \times 2$, $8 = 2 \times 4 = 2 \times 2 \times 2$, and $9 = 3 \times 3$.

6. A number is divided into factors, when any factors which will produce it are found.

ILLUSTRATIONS.—In "$12 = 6 \times 2$," 12 is divided into the factors 6 and 2, but in "$12 = 2 \times 2 \times 3$," it is divided into the factors 2, 2, and 3.

7. A number is divided into its *prime factors* when it is divided into factors which are all prime numbers.

ILLUSTRATIONS.—$18 = 2 \times 3 \times 3$. $30 = 2 \times 3 \times$

8. The product of a number taken any number of times factor, is called a POWER of that number.

ILLUSTRATIONS. — 8, which is the product of $2 \times 2 \times 2$, *i. e.*, taken 3 times as a factor is the *third power of two*; 25, which i product of 5×5, *i. e.*, of 5 taken 2 times as a factor is the *second p of five.*

9. We may indicate the power of a number by writing a s figure, called an EXPONENT, above it and a little to the right.

ILLUSTRATIONS.—$3^3 = 3 \times 3 \times 3$, or 3 to the third power. 2 $\times 2 \times 2 \times 2 \times 2$, or 2 to the fifth power.

10. The second power of a number is sometimes called *square*, and the third power its *cube*. Thus, 8, or 2^3, is the of 2; 25, or 5^2, is the *square* of 5.

B. Write the prime factors of the numbers from 1 to 10 in the following model:—

1 = Prime.

2 = Prime.

3 = Prime.

$4 = 2 \times 2 = 2^2$.

5 = Prime.

$6 = 2 \times 3$.

7 = Prime.

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$.

$10 = 2 \times 5$.

C. A factor is *common* to two or more numbers when it factor of each of them.

ILLUSTRATIONS.—2 is a common factor of 4, 8, and 14, for it factor of each.

1. What prime factors are common to 24, and 48?

Dividing each number into its prime factors, gives—

SOLUTION.—$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$.

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$.

By inspecting these, we see that 2^2 and 3 are factors of each number, and that there is no other common factor. Hence 2, 2, and 3, or 2^2 and 3 are the prime factors required.

What prime factors are common —

3. To 12 and 18?
4. To 15 and 25?
5. To 11 and 20?
6. To 30 and 40?
7. To 36 and 54?
8. To 7 and 9?
9. To 39 and 54?
10. To 6, 8, and 10?
11. To 12, 18, and 30?
12. To 20, 30, and 50?
13. To 42, 56, and 84?
14. To 63, 81, and 99?
15. To 8, 9, and 25?
16. To 7, 49, and 84?

D. A COMMON DIVISOR of two or more numbers is any number which will exactly divide each of them.

ILLUSTRATION.—4 is a common divisor of 4, 8, 12, and 32.

The GREATEST COMMON DIVISOR of two or more numbers is the largest number which is a divisor of each of them. It is also the product of all their common prime factors.

1. What is the greatest common divisor of 24, 36, and 60?

SOLUTION. — The greatest common divisor of 24, 36, and 60, is the product of all the prime factors common to these numbers.

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2.$$

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

We see that the only common prime factors are 2, 2, and 3. Hence 2 × 2 × 3, or 12, must be the greatest common divisor required.

What is the greatest common divisor —

2. Of 30 and 42?
3. Of 4 and 12?
4. Of 8 and 20?
5. Of 35 and 49?
6. Of 63 and 72?
7. Of 42 and 63?
8. Of 72 and 96?
9. Of 4, 6, and 12?
10. Of 3, 9, and 15?
11. Of 18, 27, and 45?
12. Of 14, 28, and 56?
13. Of 30, 48, and 54?
14. Of 24, 60, and 84?
15. Of 45, 75, and 90?

LESSON XLIII.

A. The product of any numbers is sometimes called their MUL TIPLE. Thus, 12 is a multiple of 1, of 2, of 3, of 4, of 6, and o 12, for it equals 1 × 12, or 2 × 6, or 3 × 4.

Hence, any number is a multiple of all its factors and divisor and a factor of all its multiples.

Every multiple of a number must contain all the prime facto of that number.

1. What prime factors must every multiple o 18 contain?

SOLUTION.—Since 18 = 2×3^2, every multiple of 18 must contai the factors 2 and 3^2, or 2, 3, and 3.

What prime factors must be contained in ever multiple —

2. Of 12?
3. Of 9?
4. Of 21?
5. Of 33?
6. Of 28?
7. Of 75?
8. Of 48?
9. Of 60?
10. Of 36?

B. A COMMON MULTIPLE of several numbers is a number whic is a multiple of all of them.

The **Least Common Multiple** of several numbers is the least number which is a multiple of all of them, and is therefore the smallest number which contains all the prime factors of each of them.

1. What is the least common multiple of 36 and 48?

Solution.—The least common multiple of 36 and 48 is the smallest number which contains all their prime factors.

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2.$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4\ 3.$$

Solution.—We must have 48 or its factors, which are $2^4 \times 3$. We must also have the factors of 36, which are $2^2 \times 3^2$, but as we have already taken $2^2 \times 3$, we have only to introduce the remaining factor 3, which gives 48×3, or $2^4 \times 3^2 = 144$, as the L. C. M required.

2. What is the least common multiple of 9, 24, and 30?

Solution.—The least common multiple of these numbers is the least number which contains all the prime factors of each of them.

$$9 = 3 \times 3.$$

$$24 = 2 \times 2 \times 2 \times 3.$$

$$30 = 2 \times 3 \times 5.$$

We must have 30 or its factors, which are $2 \times 3 \times 5$. We must also have the factors of 24, which are $2 \times 2 \times 2 \times 3$; but as we have already taken 2×3, we have only to introduce the remaining factors 2×2, which will give $2 \times 3 \times 5 \times 2 \times 2$, or $30 \times 2 \times 2$. We must have the factors of 9, which are 3 and 3, but as we have already taken one 3, we have only to introduce another 3, which gives as the L. C. M. required, $2 \times 3 \times 5 \times 2 \times 2 \times 3$, or $30 \times 2 \times 2 \times 3 = 360$.

What is the least common multiple—

3. Of 8 and 12?
4. Of 6 and 9?
5. Of 2, 4, and 6?
6. Of 9, 12, and 18?

7. Of 4 and 12?
8. Of 7 and 8?
9. Of 12 and 15?
10. Of 16 and 20?
11. Of 10, 25, and 30?
12. Of 4, 6, and 12?
13. Of 3, 4, 5, and 6?
14. Of 8, 10, 12, and 20?

Note.—If the class have time for it, the Teacher will do well give them the method explained in ARITHMETIC AND ITS APPLICATIO page 154.

LESSON XLIV.

Oral Exercise.—Exhibit any convenient thing, as an apple the class, and, cutting it into two equal parts, ask, "What h I done to the apple?" *Ans.*—"You have cut it." "Into l many parts have I cut it?" *Ans.*—"Into two parts." "How the parts compare in size?" *Ans.*—"They are equal." "T how have I divided the apple?" *Ans.* — "You have divide into two equal parts." "When anything is divided into two eq parts, the parts are called halves of the thing. What, th will you call these parts of an apple?" *Ans.* — "Halves of an apple." "What will you call one part?" *Ans.* — "One-half of an apple." "What will you call both parts?" *Ans.* — "Two halves of an apple." "What do both together equal?" *Ans.* — "A whole apple." "Then how many halves of an apple equal a whole one?" *Ans.*—"Two halves of an apple."

Continue and extend these illustrations by exercises similar in character to these suggested below: —

"How shall I divide this apple (showing another) into halves?" Dividing it, ask, "How many halves have I from it? How many halves did I have from the first apple? How many halves are there in all? Then two halves and two halves are how many halves? If I should give away one-half, how many halves should I have left? Then one-half from four halves leaves how many halves?"

Vary these exercises, dividing apples, strings, pieces of paper, lines, &c., till the class understand fully the value of halves, thirds, &c., and see clearly that they can be added, subtracted, multiplied, and divided as whole numbers are. Such a course will save much hard labor afterwards, both to Teacher and pupil.

A. EXPLANATIONS.—1. Such parts as are obtained by dividing anything or any number into two equal parts, are called halves of that thing or number. One such part is called ONE HALF; two such parts are called TWO HALVES; three such parts are called THREE HALVES, &c., &c.

2. Such parts as are obtained by dividing anything or number into 3 equal parts, are called THIRDS of the thing or number. One such part is called ONE THIRD, two such parts are called TWO THIRDS, three such parts are called THREE THIRDS, four such parts, FOUR THIRDS, &c., &c.

3d. In like manner such parts as are obtained by dividing anything or number into four equal parts are called FOURTHS of the thing or number; such as are obtained by dividing it into five equal parts, are called FIFTHS; into six, are called SIXTHS; into seven, are called SEVENTHS, &c., &c.

4. Parts like these are called FRACTIONAL PARTS.

B. 1. What are sevenths?

Ans.—Sevenths of any thing or number are such fractional parts as would be obtained by dividing it into seven equal parts.

In the same way explain each of the following:—

2. Fifths?
3. Thirds?
4. Ninths?
5. Twelfths?
6. Halves?
7. Sixths?
8. Fourths?
9. Twentieths?
10. Tenths?

C. 1. Instead of the word *sixths*, we may write the figure 6 with a line above it, thus: $\overline{6}$. Instead of the word *halves*, we may write $\overline{2}$, &c., &c.

Hence $\overline{10}$ means tenths, which are fractional parts of such kinds as are obtained by dividing a unit into 10 equal parts.

$\overline{8}$ means eighths, which are, &c.
$\overline{3}$ means thirds, which are, &c.
$\overline{21}$ means twenty-firsts, which are, &c.
$\overline{5}$ means fifths, which are, &c.

The number which thus shows into how many parts a unit is divided, is called a DENOMINATOR, because it gives a *name* or *denomination* to the parts. To indicate that a number is a denominator, draw a little line over it.

D. 1. How many sixths does it take to equal the whole of anything?

Ans.—Six, because sixths are such parts as are obtained by dividing a unit into six equal parts.

2. How many ninths does it take to equal the whole of anything?

3. How many twelfths? 6. How many $\frac{1}{20}$?
4. How many halves? 7. How many $\frac{1}{17}$?
5. How many thirds? 8. How many $\frac{1}{8}$?

E. In order to show how many fractional parts are taken or considered, a figure is written above the denominator.

Illustrations. — To express three fourths, the figure 3 may be written above the denominator 4, thus: $\frac{3}{4}$.

Five sixths may be written $\frac{5}{6}$.

Eight fifteenths may be written $\frac{8}{15}$.

In like manner, $\frac{4}{9}$ = 4 ninths; $\frac{13}{17}$ = 13 seventeenths; and $\frac{29}{31}$ = 29 thirty-firsts.

The figure which thus *enumerates* or *numbers* the parts, is called the NUMERATOR, and shows how many parts are taken or considered. The numerator is written above the denominator, and separated from it by a line.

F. Such expressions as "two-thirds," "three-elevenths," "$\frac{3}{17}$," "$\frac{5}{9}$," &c., &c., are called FRACTIONS.

1. What does the fraction $\frac{3}{4}$ express?

Ans.—The fraction three-fourths expresses the value of 3 such parts as are obtained by dividing a unit into 4 equal parts.

In the same manner explain each of the following fractions: —

2. $\frac{2}{3}$. 4. $\frac{4}{9}$. 6. $\frac{6}{7}$. 8. $\frac{16}{21}$.
3. $\frac{5}{6}$. 5. $\frac{21}{22}$. 7. $\frac{13}{15}$. 9. $\frac{23}{24}$.

G. Fractions may be explained after the following model: —

The fraction three fourths expresses the value of three equal parts of such kind that four of them would equal a unit.

1. Explain each of the fractions under F. according to t model.

2. What is the numerator and what the denominator of

H. A mixed number is a whole number and a fraction.

ILLUSTRATIONS.—$4\frac{2}{3}$, which is read "four and two-thirds."

Read each of the following:—

1. $5\frac{1}{3}$. 2. $5\frac{6}{7}$. 3. $4\frac{5}{8}$. 4. $28\frac{4}{5}$.

I. These illustrations suggest the following definitions:-

1. FRACTIONAL PARTS *of any thing, quantity or number, a parts as are obtained by dividing it into equal parts.* Or—

2. FRACTIONAL PARTS *of any thing, quantity or number, ar parts of such kind that a given number of them will equal that quantity, or number.*

3. A FRACTION expresses the value of one or more suc as are obtained by dividing a unit into equal parts. Or—

4. A FRACTION expresses the value of one or more such parts that a given number of them will equal a unit.

5. The number which shows into how many parts the t divided, or how many of the parts are equal to a unit, is the DENOMINATOR of the fraction.

6. *The number which shows how many parts are taken o dered, is called the* NUMERATOR *of the fraction.*

7. The denominator is so called because it gives the n denomination to the parts.

8. The numerator is so called because it enumerates th taken or considered.

J. Write each of the following in figures:—

1. Two-thirds.
2. Eight-ninths.
3. Thirteen-nineteenths.
4. Six twenty-firsts.
5. Four and seven-tenths.
6. Ten and four-fifths.
7. Twelve and eleven-twelfths.
8. Six and two-thirds.

LESSON XLV.

A. Fractions may arise from division as in the following examples: —

29 = * times 6?

Solution. — 29 = 4 times 6, with 5 remaining, or it equals $4\frac{5}{6}$ times 6.

Note.—The first part of the above solution should be omitted as soon as the pupil is prepared to give the final answer without it. The entire dividend is here divided, and the *fraction five sixths* is a part of the quotient, and not, like the *remainder* 5, a part of the dividend. Hence it is wrong to say "29 = 4 times 6, with $\frac{5}{6}$ remaining." These distinctions are important, and should be observed in the solutions. See Lesson XXXVI., Note under A.

Perform by the above solution the examples under Letter A. and B., Lesson XXXVI.

B. 1. How many quarts of vinegar at 6 cents per quart, can be bought for 53 cents?

Solution.—Since 1 quart of vinegar can be bought for 6 cents, as many quarts can be bought for 53 cents as there are times 6 in 53, which are $8\frac{5}{6}$ times. Hence, $8\frac{5}{6}$ quarts of vinegar at 6 cents per quart can be bought for 53 cents.

2. How many pounds of sugar at 8 cents per pound, can be bought for 68 cents?

3. How many yards of cloth at $4 per yard, can be bought for $31?

4. How many bags, each containing 3 bushels, can be from 29 bushels of grain?

5. How many hours will it take a horse to trot 33 miles, trots 7 miles per hour?

6. How many weeks will it take a man to earn $78, if he $9 per week?

7. How many hours will it take a ship to sail 63 miles, sail 8 miles per hour?

8. How many months will it take a man, who earns $1 month, to earn $105?

9. A man put 9 bu. 3 pk. of grain into bags, each hold bu. 3 pk. How many bags could he fill?

10. If a man can earn enough in one day to buy 1 gal. of oil, how many days will it take him to earn enough to b gal. 1 qt.?

LESSON XLVI.

A. Fractions may be added, subtracted, multiplied, and di as whole numbers are. Thus:—

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$, just as 2 days + 4 days = 6 days.

$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$, just as 5 qts. — 3 quarts = 2 quarts.

9 times $\frac{3}{4} = \frac{27}{4}$, just as 9 times 3 pecks = 27 pecks.

$\frac{6}{25}$ are contained 4 times in $\frac{24}{25}$, just as 6 lb. are contai times in 24 lb.

B. 1. 7 = * fourths?

Solution. — Since 1 = 4 fourths, 7 must equal 7 times 4 fo which are 28 fourths. Therefore, 7 = $\frac{28}{4}$.

2. $8\frac{4}{5}$ = * fifths ?

Solution. — Since 1 = 5 fifths, 8 must equal 8 times 5 fifths, which are 40 fifths, and 4 fifths added are 44 fifths. Hence $8\frac{4}{5} = \frac{44}{5}$.

Note.—Compare these solutions with those of E., Lesson XXX

3. 9 = * tenths ?
4. 5 = * thirds ?
5. 8 = * nineteenths ?
6. $4\frac{3}{4}$ = * fourths ?
7. $4\frac{3}{7}$ = * seventeenths ?
8. $2\frac{8}{19}$ = * nineteenths ?
9. $7\frac{13}{17}$ = * seventeenths ?
10. $6\frac{9}{11}$ = * elevenths ?

C. 1. $\frac{58}{9}$ = * ones ?

Solution.—Since 9 ninths = 1, 58 ninths must equal as many ones as there are times 9 in 58, which are $6\frac{4}{9}$ times. Hence, $\frac{58}{9} = 6\frac{4}{9}$.

Note.—Compare this solution with those of F., Lesson XXX.

2. $\frac{64}{8}$ = * ones ?
3. $\frac{27}{5}$ = * ones ?
4. $\frac{91}{7}$ = * ones ?
5. $\frac{33}{4}$ = * ones ?
6. $\frac{67}{9}$ = * ones ?
7. $\frac{46}{7}$ = * ones ?
8. $\frac{87}{15}$ = * ones ?
9. $\frac{184}{13}$ = * ones ?
10. $\frac{36}{7}$ = * ones ?
11. $\frac{94}{12}$ = * ones ?

D. 1. What is the sum of $\frac{9}{11} + \frac{7}{11}$?

1st **Solution.**—$\frac{9}{11} + \frac{7}{11} = \frac{16}{11} = 1\frac{5}{11}$

2d **Solution.** — Observing that $\frac{9}{11} + \frac{2}{11} = \frac{11}{11}$, or 1, we may have $\frac{9}{11} + \frac{7}{11} = \frac{9}{11} + \frac{2}{11} + \frac{5}{11} = 1\frac{5}{11}$.

Note.—The second forms of solution to the problems under D. and G., and the third form under E., will often be found much easier than the first.

2. $\frac{7}{9} + \frac{4}{9}$?
3. $\frac{9}{13} + \frac{11}{13}$?
4. $\frac{7}{12} + \frac{11}{12}$?
5. $\frac{3}{7} + \frac{6}{7} + \frac{4}{7} + \frac{2}{7}$?
6. $\frac{7}{8} + \frac{6}{8} + \frac{4}{8} + \frac{5}{8}$?
7. $\frac{8}{12} + \frac{9}{12} + \frac{7}{12} + \frac{11}{12}$?

E. 1. $4 + 3 + \frac{7}{8}$? Then $4 + 3\frac{7}{8}$?
2. $8 + 7 + \frac{3}{5}$? Then $8 + 7\frac{3}{5}$?
3. $12 + 21 + \frac{3}{5}$? Then $12 + 21\frac{3}{5}$?

4. What is the sum of $6\frac{4}{5} + 8\frac{3}{5}$?

1st Solution.—$6\frac{4}{5}$ and 8 are $14\frac{4}{5}$, and $\frac{3}{5}$ are $14\frac{7}{5} = 15\frac{2}{5}$

2d Solution.—$6 + 8 = 14$; $\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$, which, added to $14 = 15\frac{2}{5}$.

3d Solution.—Observing that $6\frac{4}{5} + \frac{1}{5} = 7$, we have $6\frac{4}{5} + 8\frac{3}{5} = 6\frac{4}{5} + \frac{1}{5} + 8\frac{2}{5} = 7 + 8\frac{2}{5} = 15\frac{2}{5}$.

5. $9\frac{4}{7} + 6\frac{5}{7}$?
6. $3\frac{3}{5} + 4\frac{4}{5}$?
7. $4\frac{8}{11} + 3\frac{6}{11}$?
8. $5\frac{17}{25} + 8\frac{18}{25}$?
9. $8\frac{9}{16} + 7\frac{5}{16} + 6\frac{9}{16}$?
10. $5\frac{2}{3} + 7\frac{2}{3} + 4\frac{1}{3} + 5\frac{2}{3}$?
11. $8\frac{4}{5} + 2\frac{3}{5} + 7\frac{4}{5} + 3\frac{3}{5}$?
12. $5\frac{7}{9} + 6\frac{4}{9} + 5\frac{7}{9} + 4\frac{6}{9}$?

F. 1. $6\frac{7}{8} - 3\frac{5}{8}$?

Solution.—$6\frac{7}{8} - 3 = 3\frac{7}{8}$; $3\frac{7}{8} - \frac{5}{8} = 3\frac{2}{8}$. Hence, $6\frac{7}{8} - 3\frac{5}{8} = 3\frac{2}{8}$.

2. $8\frac{6}{7} - 2\frac{4}{7}$?
3. $14\frac{8}{9} - 7\frac{4}{9}$?
4. $18\frac{7}{12} - 4\frac{5}{12}$?
5. $16\frac{4}{5} - 9\frac{3}{5}$?
6. $15\frac{13}{21} - 4\frac{8}{21}$?
7. $38\frac{43}{73} - 29\frac{37}{73}$?

G. 1. What is the value of $1 - \frac{7}{12}$?

Solution.—$1 = \frac{12}{12}$, and $\frac{12}{12} - \frac{7}{12} = \frac{5}{12}$.

2. What is the value of $8 - \frac{9}{20}$?

SOLUTION. — $8 - \frac{9}{20} = 7\frac{20}{20} - \frac{9}{20} = 7\frac{11}{20}$

3. $1 - \frac{3}{5}$?
4. $1 - \frac{13}{25}$?
5. $1 - \frac{17}{25}$?
6. $3 - \frac{5}{7}$?
7. $8 - \frac{3}{11}$?
8. $9 - \frac{5}{8}$?

9. $8\frac{7}{20} - \frac{13}{20}$?

1st SOLUTION.—$8\frac{7}{20} = 7 + 1\frac{7}{20} - \frac{13}{20} = 7\frac{27}{20} - \frac{13}{20} = 7\frac{14}{20}$.

2d SOLUTION. —$8\frac{7}{20} - \frac{13}{20} = 8\frac{7}{20} - \frac{7}{20} - \frac{6}{20} = 8 - \frac{6}{20} = 7\frac{14}{20}$.

10. $23\frac{5}{13} - 16\frac{9}{13}$?

1ST SOLUTION.—$23\frac{5}{13} - 16 = 7\frac{5}{13} = 6\frac{18}{13}$; $6\frac{18}{13} - \frac{9}{13} = 6\frac{9}{13}$.

2d SOLUTION. — $23\frac{5}{13} - 16 = 7\frac{5}{13}$; $7\frac{5}{13} - \frac{9}{13} = 7\frac{5}{13} - \frac{5}{13} - \frac{4}{13} = 7 - \frac{4}{13} = 6\frac{9}{13}$.

11. $9\frac{1}{6} - 7\frac{5}{6}$?
12. $4\frac{3}{16} - \frac{9}{16}$?
13. $16\frac{4}{12} - 5\frac{7}{12}$?
14. $43\frac{2}{5} - 17\frac{4}{5}$?
15. $23\frac{4}{11} - 13\frac{9}{11}$?
16. $8\frac{2}{17} - 3\frac{14}{17}$?
17. $64\frac{2}{3} + 4\frac{2}{3} - 8\frac{2}{3}$?
18. $23\frac{3}{4} + 17\frac{2}{4} - 8\frac{3}{4}$?

H. 1. George had a very large apple. He gave William $\frac{2}{5}$ of it, Joseph $\frac{2}{5}$ of it, and ate the rest. What part of it did he eat?

2. Edward earned $\frac{3}{4}$ of a dollar by picking blackberries, $\frac{3}{4}$ of a dollar by picking strawberries, and $\frac{3}{4}$ of a dollar by picking blueberries. How much did he earn in all?

3. Who can tell whether the sum of $8\frac{2}{5} + 5\frac{4}{5} + 8\frac{3}{5} + 7\frac{3}{5}$ is greater or less than $24\frac{4}{5}$, and how much?

4. From a lot containing $8\frac{4}{9}$ acres, there were $5\frac{8}{9}$ acres sold. How many acres were left?

5. Mr. Stone gave $\frac{5}{12}$ of his money for a lot of land, and $\frac{5}{12}$ for a horse. What part of it had he left?

6. Isaac caught three nice trout. The first weighed $3\frac{7}{16}$ lb., the second weighed $2\frac{14}{16}$ lb., and the third weighed $1\frac{15}{16}$ lb. How much did they all weigh?

7. Mr. Davis owns $\frac{7}{20}$ of a vessel, Mr. Mason owns $\frac{6}{20}$, and Mr. Allen owns the rest. What part of the vessel does Mr. Allen own?

8. Julia's father gave her $\frac{23}{35}$ of a dollar, her mother gave her $\frac{19}{35}$, her brother gave her $\frac{12}{35}$, and her uncle gave her enough to make up 2 dollars. How much did her uncle give her?

9. Farmer Brown had $17\frac{4}{23}$ tons of hay in his barn, and $15\frac{7}{23}$ tons in his stacks. How many tons had he in both? He moved $8\frac{15}{23}$ tons from his stacks into his barn. How many tons were then in his barn? How many in his stacks?

10. A man paid $11\frac{3}{8}$ dollars for a coat, $8\frac{7}{8}$ dollars for a pair of pants, and $2\frac{5}{8}$ dollars for a vest, giving in payment a twenty-dollar bill. How much ought he to receive back?

LESSON XLVII.

A. 1. 9 times $\frac{6}{7}$?

SOLUTION.—9 times $\frac{6}{7} = \frac{54}{7}$, which, since $\frac{7}{7} = 1$, must equal as many ones as there are times 7 in 54, which are $7\frac{5}{7}$ times. Hence, 9 times $\frac{6}{7} = 7\frac{5}{7}$.

ABBREVIATED SOLUTION.—9 times $\frac{6}{7} = \frac{54}{7} = 7\frac{5}{7}$.

2. 4 times $\frac{9}{12}$?

3. 8 times $\frac{4}{5}$?

4. 4 times $\frac{8}{9}$?

5. 7 times $\frac{11}{12}$?

6. 9 times $\frac{7}{8}$?

7. 4 times $\frac{9}{12}$?

8. 5 times $\frac{8}{9}$?

9. 6 times $\frac{11}{13}$?

10. 6 times $7\frac{3}{5}$?

Solution.—6 times 7 = 42, and 6 times $\frac{3}{5} = \frac{18}{5} = 3\frac{3}{5}$, which, added to 42, gives $45\frac{3}{5}$. Hence, 6 times $7\frac{3}{5} = 45\frac{3}{5}$.

11. 9 times $8\frac{2}{3}$?
12. 8 times $9\frac{3}{4}$?
13. 7 times $4\frac{3}{5}$?
14. 4 times $6\frac{13}{15}$?
15. 6 times $8\frac{9}{12}$?
16. 5 times $9\frac{5}{7}$?
17. 4 times $8\frac{11}{12}$?
18. 6 times $14\frac{9}{10}$?

B. 1. $12\frac{3}{4}$ = * times $2\frac{1}{4}$?

Solution.—$12\frac{3}{4} = \frac{51}{4}$; $2\frac{1}{4} = \frac{9}{4}$; and $\frac{51}{4}$ contains $\frac{9}{4}$ as many times as 51 contains 9, which are $5\frac{6}{9}$ times. Hence, $12\frac{3}{4} = 5\frac{6}{9}$ times $2\frac{1}{4}$.

2. $8\frac{2}{3} \div 3$?

Solution.—$8\frac{2}{3} = \frac{26}{3}$; $3 = \frac{9}{3}$; and $\frac{26}{3} \div \frac{9}{3} = 26 \div 9 = 2\frac{8}{9}$. Hence $8\frac{2}{3} \div 3 = 2\frac{8}{9}$.

3. $7\frac{1}{2}$ = * times $1\frac{1}{2}$?
4. $4\frac{2}{3}$ = * times $1\frac{2}{3}$?
5. $8\frac{4}{7}$ = * times $1\frac{4}{7}$?
6. $9\frac{3}{5}$ = * times $2\frac{4}{5}$?
7. $5\frac{4}{5}$ = * times $\frac{7}{8}$?
8. $7\frac{1}{3} \div 1\frac{4}{5}$?
9. $8\frac{3}{4} \div 2\frac{1}{4}$?
10. $9\frac{1}{2} \div 3\frac{1}{2}$?
11. $6\frac{3}{7} \div 1\frac{2}{7}$?
12. $8\frac{3}{4} \div 1\frac{3}{4}$?

C. 1. How much will 7 yards of cloth cost at $9\frac{3}{4}$ dollars per yard.

2. A man gave 6 cords of wood at $\$4\frac{2}{3}$ per cord for raisins at \$7 per cask. How many casks did he buy?

3. How many yards of cloth, at \$8 per yard, can be bought for 8 bbls. of cider at $\$3\frac{3}{4}$ per barrel?

4. I gave 9 firkins of butter at \$$4\frac{4}{6}$ per firkin for flour at \$7 per barrel. How many barrels did I buy?

5. How many shade trees, worth $\frac{4}{7}$ of a dollar a-piece, can be bought for $5\frac{1}{7}$ dollars?

6. How many skeins of silk, worth $\frac{2}{3}$ of a dime per skein, can be bought for $6\frac{2}{3}$ dimes?

7. How many baskets, each containing $\frac{6}{7}$ of a bushel, can be filled from $8\frac{3}{7}$ bushels of peaches?

8. How many boxes, each holding $\frac{7}{8}$ of a quart, can be filled from $7\frac{5}{8}$ quarts of blackberries?

9. A man paid $6\frac{2}{5}$ dollars for grain at $\frac{4}{5}$ of a dollar per bushel. How many bushels did he buy? He put the grain into bags each holding $1\frac{1}{3}$ bushels. How many bags did he fill?

10. Ralph paid $5\frac{2}{5}$ dimes for parched corn at $\frac{3}{5}$ of a dime per quart. How many quarts did he buy? After giving away $\frac{1}{5}$ of a quart, he put the rest into paper bags each holding $\frac{4}{5}$ of a quart. How many bags did it take?

11. A farmer exchanged 5 barrels of apples at $1\frac{6}{7}$ dollars per barrel, for oil at $1\frac{1}{7}$ dollars per gallon. How many gallons of oil did he get?

12. How many pounds of tea, worth $\frac{5}{8}$ of a dollar per pound, should be given for three books worth $2\frac{1}{2}$ dollars a-piece?

13. A man who had $10\frac{3}{8}$ bushels of potatoes, used $2\frac{3}{8}$ bushels, and then sold the rest at $\frac{7}{8}$ of a dollar per bushel, receiving in payment cloth at $\frac{3}{8}$ of a dollar per yard. How many yards of cloth did he receive?

14. I bought 8 barrels of flour at \$$7\frac{4}{5}$ per barrel, and gave in payment 12 cords of wood at \$$4\frac{3}{5}$ per cord, and the rest in apples at $\frac{4}{5}$ of a dollar per bushel. How many bushels of apples did I give?

LESSON XLVIII.

A. 1. What part of 5 is 1?

Ans.—1 is $\frac{1}{5}$ of 5, because 5 times 1 = 5.

What part —

2. Of 7 is 1?
3. Of 2 is 1?
4. Of 9 is 1?
5. Of 10 is 1?
6. Of 8 is 1?
7. Of 3 is 1?

B. 1. What part of 8 is 3?

Solution.—Since 1 = $\frac{1}{8}$ of 8, 3 must equal $\frac{3}{8}$ of 8.

What part —

2. Of 12 is 7?
3. Of 9 is 4?
4. Of 7 is 2?
5. Of 10 is 9?
6. Of 9 is 10?
7. Of 13 is 11?
8. Of 8 is 5?
9. Of 5 is 8?

C. 1. What part of 9 quarts is 4 quarts?

Solution.—4 quarts is the same part of 9 quarts that 4 is of 9, which is $\frac{4}{9}$.

What part —

2. Of 8 yd. is 3 yd.?
3. Of 11 lb. is 7 lb.?
4. Of 7 lb. is 11 lb.?
5. Of $5 is $3?
6. Of £12 is £5?
7. Of £5 is £12?

8. What part of the cost of 7 yd. is the cost of 4 yd.?
9. What part of the cost of 3 acres is the cost of 4 acres?

10. What part of the cost of 10 bushels is the cost of 9 bushels?

11. When flour is 9 dollars per barrel, what part of a barrel can be bought for 1 dollar? For 5 dollars?

12. Mr. Edwards and Mr. Boyden bought a cask of oil, containing 8 gallons, which they so divided that Mr. Edwards had 3 gallons, and Mr. Boyden had 5. What part of the cost should each pay?

13. Mr. Avery, Mr. Leavens, and Mr. Congdon together bought 17 bushels of corn, which they so divided that Mr. Avery took 4 bushels, Mr. Leavens took 6 bushels, and Mr. Congdon took the remainder. What part of the cost should each pay?

LESSON XLIX.

A. 1. What is $\frac{1}{9}$ of 63?

Ans.—$\frac{1}{9}$ of 63 is 7, because 9 times 7 = 63.

2. $\frac{1}{3}$ of 24?
3. $\frac{1}{5}$ of 25?
4. $\frac{1}{7}$ of 49?
5. $\frac{1}{6}$ of 42?
6. $\frac{1}{8}$ of 64?
7. $\frac{1}{9}$ of 81?

8. What is $\frac{3}{8}$ of 72?

1st Solution.—$\frac{3}{8}$ of 72 = 3 times $\frac{1}{8}$ of 72; $\frac{1}{8}$ of 72 is 9, and 3 times 9 = 27. Hence $\frac{3}{8}$ of 72 = 27.

2d Solution.—$\frac{1}{8}$ of 72 = 9, and $\frac{3}{8}$ of 72 must equal 3 times 9, which are 27. Hence, $\frac{3}{8}$ of 72 = 27.

9. $\frac{4}{5}$ of 40?
10. $\frac{2}{3}$ of 18?
11. $\frac{5}{8}$ of 48?
12. $\frac{7}{8}$ of 56?
13. $\frac{9}{10}$ of 80?
14. $\frac{7}{9}$ of 72?
15. $\frac{5}{9}$ of 36?
16. $\frac{8}{9}$ of 63?
17. $\frac{5}{6}$ of 54?

B. 1. How many are $4\frac{2}{3}$ times 9?

1st Solution.—$4\frac{2}{3}$ times 9 = 4 times 9 + $\frac{2}{3}$ of 9 = 4 times 9 + 2 times $\frac{1}{3}$ of 9. 4 times 9 = 36; $\frac{1}{3}$ of 9 = 3, and 2 times 3 = 6, which, added to 36 = 42. Hence, $4\frac{2}{3}$ times 9 = 42.

2d Solution.—4 times 9 = 36. $\frac{1}{3}$ of 9 = 3, and $\frac{2}{3}$ of 9 must equal 2 times 3, or 6, which, added to 36 = 42. Hence, $4\frac{2}{3}$ times 9 = 42.

2. $7\frac{1}{3}$ times 6?
3. $9\frac{3}{4}$ times 8?
4. $8\frac{4}{5}$ times 10?
5. $6\frac{2}{3}$ times 9?
6. $8\frac{3}{4}$ times 12?
7. $5\frac{6}{7}$ times 14?
8. $\frac{5}{9}$ of 54 = * times 6?
9. $\frac{6}{7}$ of 42 = * times 9?
10. $\frac{3}{5}$ of 40 = * times 7?
11. $8\frac{2}{3}$ times 9 = * times 10?
12. $5\frac{3}{4}$ times 12 = * times 8?
13. $7\frac{1}{2}$ times 8 = * times 5?
14. $\frac{4}{5}$ of 30 + $\frac{2}{7}$ of 56 = * times 5?
15. $\frac{5}{9}$ of 45 + $\frac{4}{5}$ of 25 = * times 9?
16. $\frac{4}{7}$ of 49 + $\frac{2}{9}$ of 36 = * times 6?
17. $\frac{3}{8}$ of 72 + $\frac{3}{7}$ of 63 = * times 9?
18. $\frac{2}{9}$ of 54 + $\frac{4}{7}$ of 28 = * times 7?

C. 1. If 8 pictures cost 72 cents, how much will 5 cost?

Solution.—If 8 pictures cost 72 cents, 1 picture will cost $\frac{1}{8}$ of 72 cents, or 9 cents, and 5 pictures will cost 5 times 9 cents, or 45 cents. Hence, if 8 pictures cost 72 cents, 5 will cost 45 cents.

2. If 7 sheep cost $49, how many dollars will 8 sheep cost?

3. If 8 papers of candy cost 56 cents, how much will 5 papers cost?

4. If a girl receives 45 merit-marks for 9 perfect lessons, how many will she receive for 5 perfect lessons?

5. If 8 milk cans will hold 24 quarts of milk, how many quarts will 7 milk cans hold?

6. How much will 8 quarts of molasses cost at 32 cents per gallon?

Solution. — If 1 gal. or 4 qt. cost 32 cents, 1 quart, or ¼ of a will cost ¼ of 32 cents, or 8 cents, and 3 quarts will cost 3 times 8 c or 24 cents. Therefore, 3 quarts of molasses, at 32 cents per ga would cost 24 cents.

7. How much will 7 qt. of meal cost at 24 cents per pk. ?

8. How much will 8 gills of oil cost at 48 cents per qt. ?

9. If a yard of cloth is worth 24 cents, how much is a pi feet in length worth ?

10. If 1 acre 3 roods (or 7 roods) of land cost 63 dollars, many dollars will 1 acre cost ?

11. If 1 gal. 1 qt. of burning fluid cost 81 cents, how m will 1 qt. cost ? How much will 1 gallon cost ?

12. If a pound of coffee cost 42 cents, what will 6/7 of a p cost ?

Solution.—If a pound of coffee cost 42 cents, 1/7 of a pound will 1/7 of 42 cents, which is 6 cents, and 6/7 of a pound will cost 6 tim cents, or 36 cents. Therefore, 6/7 of a pound of coffee, at 42 cent pound, will cost 36 cents.

13. If a man can perform a piece of work in 72 minutes, in many minutes could he perform ⅝ of it ?

14. What will ⅚ of a yard of linen cost at 54 cents per yar

15. Brass is composed of copper and zinc. If ⅕ of it is and the rest copper, how many pounds of copper will there b a bar of brass weighing 25 lbs. ?

16. How much will 5¾ lb. of sugar cost at 8 cents per lb.

17. How much will 8⅔ barrels of flour cost at 6 dollars barrel ?

18. How many square rods of land are there in a piece 9 long and 6¾ rods wide ?

Solution.—Since a piece of land 1 rod long and 1 rod wide con 1 sq. rd., a piece 9 rods long and 1 rod wide must contain 9 sq and a piece 9 rods long and 6¾ rods wide must contain 6¾ times rd., which equal 60 sq. rd. Hence, a piece of land 9 rods long and 6⅔ rods wide contains 50 sq. rd.

19. How many sq. ft. are there in a blackboard 12 ft. long and 2⅚ ft. wide?

20. How many sq. yd. are there in a floor 5 yd. long and 4⅘ yd. wide?

21. How much will it cost to paint a surface 8 ft. long and 4⅓ wide, at 3 cents per sq. ft.?

22. How much will it cost to plaster a wall 12 ft. long and 8¾ ft. wide, at 4 cents per sq. ft.?

23. If a man can walk 32 miles in 8 hours, how far can he walk in one hour? How far in 9¾ hours?

24. If 8 tons of meadow-hay cost 72 dollars, how much will 5⅔ tons cost?

25. If 7 yd. of cloth are worth 49 lb. of butter, how many pounds of butter ought 4$\frac{6}{7}$ yd. of cloth to be worth?

26. Olive has 40 picture-books, Ella has ⅘ as many as Olive, and Ada has ⅞ as many as Ella. How many has Ada? How many has Ella?

27. John is ¼ as old as his father, who is 36 years old, and William is ⅔ as old as John. How old is John? How old is William?

28. Edward had 3 cents, and Robert had 5. They put their money together and bought 72 filberts. How many filberts ought each to have?

29. Harriet, Maria, and Caroline sent some berries to market, for which they received 63 cents. Now, if Harriet sent 3 quarts, Maria 2, and Caroline 4, how many cents ought each to receive?

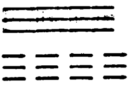
30. If for two three-cent pieces and 2 cents 64 marbles can be bought, how many marbles can be bought for 1 three-cent piece and 2 cents? How many for 3 three-cent pieces?

81. Julia's basket holds 3 pt. 1 gill. If she can fill it with berries in 45 minutes, how many minutes would it take her to fill Susan's basket, which holds but 1 pint? How long to fill Josephine's basket, which holds but 1 gill?

LESSON L.

A. 1. What is $\frac{1}{4}$ of 3?

Solution.—$\frac{1}{4}$ of 3 is $\frac{3}{4}$ of 1, for if 3 equal things should be divided into 4 equal parts, one of those parts would equal $\frac{3}{4}$ of one thing.



Note.—This may be illustrated to the eye by taking 3 equal lines and dividing them into 4 equal parts, arranged as in the figure at the left. One part will then contain $\frac{1}{4}$ of 3 lines, which, as will be seen, is equivalent to $\frac{3}{4}$ of a line.

2. $\frac{1}{5}$ of 7?
3. $\frac{1}{5}$ of 3?
4. $\frac{1}{5}$ of 2?
5. $\frac{1}{4}$ of 1?
6. $\frac{1}{5}$ of 5?
7. $\frac{1}{5}$ of 4?

B. From the preceding exercises, it follows that $\frac{3}{5}$, or $\frac{3}{5}$ of 1 $=$ $\frac{1}{5}$ of 3; that $\frac{7}{5}$, or $\frac{7}{5}$ of 1 $=$ $\frac{1}{5}$ of 7, &c. Hence, $\frac{3}{5}$ of any number equal 3 times $\frac{1}{5}$ of that number, and also $\frac{1}{5}$ of 3 times that number; $\frac{4}{5}$ of any number equal 4 times $\frac{1}{5}$ of that number, and also $\frac{1}{5}$ of 4 times that number.

1. What is $\frac{7}{8}$ of 5?

1st **Solution.**—$\frac{7}{8}$ of 5 $=$ 7 times $\frac{1}{8}$ of 5; $\frac{1}{8}$ of 5 $=$ $\frac{5}{8}$, and 7 times $\frac{5}{8}$ $=$ $\frac{35}{8}$ $=$ $4\frac{3}{8}$. Hence $\frac{7}{8}$ of 5 $=$ $4\frac{3}{8}$.

2d **Solution.**—$\frac{7}{8}$ of 5 $=$ $\frac{1}{8}$ of 7 times 5; 7 times 5 $=$ 35, and $\frac{1}{8}$ of 35 $=$ $4\frac{3}{8}$. Hence, $\frac{7}{8}$ of 5 $=$ $4\frac{3}{8}$.

Note.—The pupil should master the first solution, and then the second; and afterwards be required to use in each example the one best adapted to that example.

2. $\frac{3}{4}$ of 2? 4. $\frac{2}{5}$ of 8? 6. $\frac{6}{11}$ of 7?

3. $\frac{5}{7}$ of 6? 5. $\frac{11}{12}$ of 8? 7. $\frac{3}{5}$ of 4?

C. 1. What is $\frac{1}{8}$ of 67?

1st Solution.—$\frac{1}{8}$ of 67 = $\frac{1}{8}$ of 64 + $\frac{1}{8}$ of 3; $\frac{1}{8}$ of 64 = 8; $\frac{1}{8}$ of 3 = $\frac{3}{8}$, which, added to 8 = $8\frac{3}{8}$. Hence, $\frac{1}{8}$ of 67 = $8\frac{3}{8}$.

2d. Solution.—$\frac{1}{8}$ of 67 = $\frac{1}{8}$ of 64 + $\frac{1}{8}$ of 3 = $8\frac{3}{8}$.

3d Solution.—$\frac{1}{8}$ of 67 = $8\frac{3}{8}$.

Note.—The first and second solutions are chiefly valuable as a preparation for the third.

2. $\frac{1}{9}$ of 48? 5. $\frac{3}{4}$ of 17? 8. $\frac{7}{15}$ of 89?

3. $\frac{1}{6}$ of 28? 6. $\frac{4}{7}$ of 20? 9. $\frac{4}{5}$ of 48?

4. $\frac{1}{4}$ of 17? 7. $\frac{5}{6}$ of 80? 10. $\frac{3}{4}$ of 27?

D. 1. $\frac{1}{6}$ of $52\frac{2}{7}$?

Solution.—$\frac{1}{6}$ of $52\frac{2}{7}$ = $\frac{1}{6}$ of 48 + $\frac{1}{6}$ of $4\frac{2}{7}$; $\frac{1}{6}$ of 48 = 8; $\frac{1}{6}$ of $4\frac{2}{7}$ or of $\frac{30}{7}$ = $\frac{5}{7}$, which, added to 8 = $8\frac{5}{7}$. Hence, $\frac{1}{6}$ of $52\frac{2}{7}$ = $8\frac{5}{7}$.

2. $\frac{1}{5}$ of $17\frac{1}{2}$? 8. $\frac{1}{3}$ of 17 qt. 1 pt.?

3. $\frac{1}{9}$ of $41\frac{5}{8}$? 9. $\frac{1}{9}$ of 41 pk. 5 qt.?

4. $\frac{1}{7}$ of $66\frac{3}{5}$? 10. $\frac{1}{7}$ of 66 sq. yd. 8 sq. ft.?

5. $\frac{3}{4}$ of $26\frac{2}{3}$? 11. $\frac{4}{9}$ of 49 bu. 2 pk.?

6. $\frac{8}{9}$ of $86\frac{1}{4}$? 12. $\frac{7}{8}$ of 58 yd. 2 ft.?

7. $\frac{5}{8}$ of 75? 13. $\frac{2}{3}$ of 26 wk. 4 days?

E. 1. $\frac{7}{9}$ of 36 = * times 5? 4. $\frac{2}{7}$ of 49 = * times 8?

2. $\frac{3}{5}$ of 25 = * times 2? 5. $\frac{4}{5}$ of 45 = * times 8?

3. $\frac{7}{10}$ of 70 = * times 5? 6. $\frac{3}{9}$ of 63 = * times 6?

F. 1. $\frac{8}{9}$ of 45 = * times $\frac{1}{6}$ of 42?

Solution. — $\frac{8}{9}$ of 45 = 8 times $\frac{1}{9}$ of 45; $\frac{1}{9}$ of 45 = 5, and 8 tir
5 = 40; $\frac{1}{6}$ of 42 = 7, and 7 is contained in 40, $5\frac{5}{7}$ times. Therefo
$\frac{8}{9}$ of 45 = $5\frac{5}{7}$ times $\frac{1}{6}$ of 42.

2. $\frac{7}{10}$ of 80 = * times $\frac{1}{8}$ of 64?
3. $\frac{5}{9}$ of 36 = * times $\frac{1}{3}$ of 24?
4. $\frac{6}{8}$ of 72 = * times $\frac{1}{4}$ of 32?
5. $\frac{9}{5}$ of 40 = * times $\frac{5}{6}$ of 12?
6. $\frac{5}{2}$ of 16 = * times $\frac{2}{3}$ of 9?
7. $\frac{3}{7}$ of 25 = * times $\frac{1}{7}$ of 9?
8. $\frac{3}{5}$ of 19 = * times $\frac{1}{3}$ of 7?
9. $\frac{4}{5}$ of 14 = * times $\frac{1}{3}$ of 8?

G. 1. If 7 inkstands cost 45 cents, what will 3 cost?

2. If 9 melons cost 77 cents, what will 5 cost?

3. If 3 weeks' board cost $27, what will 7 cost?

4. If 4 men eat 23 pounds of meat in a month, how ma
pounds will 7 men eat in the same time?

5. If 8 horses eat 37 cwt. of hay in a month, how much wil
horses eat in the same time?

6. If 1 pk. of cranberries cost 53 cents, how many cwt. wi
qt. cost?

7. If a gallon of burning fluid is worth 79 cents, what are 1
1 pt. worth?

8. $\frac{5}{9}$ of a furlong = how many rods?

Suggestion.—Since 40 rods = 1 furlong, $\frac{5}{9}$ of a furlong equal
40 rods.

9. $\frac{5}{9}$ of a qr. = how many pounds?

10. $\frac{6}{7}$ of a cu. yd. = how many cubic feet?

11. $\frac{5}{9}$ of an hour = how many minutes?

12. $\frac{4}{9}$ of a day = how many hours?

13. $\frac{2}{3}$ of a bu. = how many pk., qt., &c.?

SOLUTION.—Since 1 bu. = 4 pk., $\frac{2}{3}$ of a bu. must equal $\frac{2}{3}$ of 4 pk., or $2\frac{2}{3}$ pk. But since 1 pk. = 8 qt., $\frac{2}{3}$ of a pk. must equal $\frac{2}{3}$ of 8 qt., or $5\frac{1}{3}$ qt. Since 1 qt. = 2 pt., $\frac{1}{3}$ of a qt. must equal $\frac{1}{3}$ of 2 pt., or $\frac{2}{3}$ of a pt. Since 1 pt. = 4 gills, $\frac{2}{3}$ of a pt. must equal $\frac{2}{3}$ of 4 gills, or $2\frac{2}{3}$ gills. Therefore, $\frac{2}{3}$ of a bu. = 2 pk. 5 qt. 0 pt. $2\frac{2}{3}$ gills.

14. $\frac{5}{7}$ of a £ = how many s., d., and qr.?

15. $\frac{4}{5}$ of a lb. = how many oz. and dwt.?

16. $\frac{6}{7}$ of a ton = how many cwt., qr., lb.?

17. $\frac{5}{8}$ of a sq. yd. = how many sq. ft., sq. in.?

18. $\frac{5}{7}$ of a lb. = how many oz., dwt., gr.?

19. $\frac{4}{9}$ of a wk. = how many da., h., &c.?

20. $\frac{3}{5}$ of a lb. = how many ʒ., ℥., &c.?

21. $\frac{7}{9}$ of a £. = how many s., d. qr.?

22. Frederic and Benjamin gathered some nuts, of which Frederick gathered 4 qts., and Benjamin 2 qts. They sold them for 39 cents. How many cents ought each to receive?

23. Mr. Ames and Mr. Clapp bought the apples on 2 large trees for 9 dollars. Mr. Ames paid 5 dollars, and Mr. Clapp paid 4 dollars. There proved to be 37 pecks of apples on the trees. How many pecks ought each to have? How many bushels?

24. If 4 pounds of rice are worth 24 cents, how many pounds of rice ought to be given for $7\frac{3}{4}$ pounds of sugar worth 8 cents per pound?

25. How many half-pounds of coffee, worth 16 cents per pound, would be given for $2\frac{1}{4}$ bushels of oats, worth 44 cents per bushel?

26. I gave 4 pk. 3 qt. of nuts, worth 24 cents per peck, for eggs at 12 cents per dozen. How many dozen eggs did I receive?

27. I worked $3\frac{1}{6}$ weeks, at $18 per week, and received in p
ment $30 in money and the balance in shoes at $3 per p
How many pairs of shoes did I receive?

28. James asked his father for some drawing-pencils, to wh
his father replied, "4 good drawing-pencils will cost as much
3 writing books at $1 per dozen. Now, if you will tell me
much 6 drawing-pencils will cost, I will buy them for yo
What should have been James's answer?

LESSON LI.

A. 1. What part of $\frac{5}{7}$ is $\frac{2}{7}$?

SOLUTION.—$\frac{2}{7}$ is the same part of $\frac{5}{7}$ that 2 is of 5, which is $\frac{2}{5}$.

What part —

2. Of $\frac{4}{9}$ is $\frac{7}{9}$? 4. Of $\frac{4}{21}$ is $\frac{1}{21}$? 6. Of $\frac{6}{11}$ is $\frac{3}{11}$
3. Of $\frac{6}{11}$ is $\frac{5}{11}$? 5. Of $\frac{1}{3}$ is $\frac{4}{3}$? 7. Of $\frac{5}{4}$ is $\frac{3}{4}$?

8. What part of $2\frac{2}{3}$ is $10\frac{1}{3}$?

SOLUTION.—$2\frac{2}{3} = \frac{8}{3}$; $10\frac{1}{3}$, and $\frac{31}{3}$ is the same part of $\frac{8}{3}$ that 31 is
which is $\frac{31}{8}$ or $3\frac{7}{8}$. Hence, $10\frac{1}{3} = \frac{31}{8}$ of $2\frac{2}{3}$, or it equals $3\frac{7}{8}$ times

What part —

9. Of $2\frac{1}{2}$ is $4\frac{1}{2}$? 14. Of 3 da. is 1 wk.?
10. Of $7\frac{1}{7}$ is $2\frac{2}{7}$? 15. Of 4 sq. ft. is 1 sq. yd.
11. Of $9\frac{3}{5}$ is $2\frac{4}{5}$? 16. Of 1 pk. 1 qt. is 3 pk. 7 q
12. Of $\frac{4}{5}$ is 1? 17. Of 3 yd. 1 ft. is 8 yd. 2 ft.
13. Of $\frac{2}{3}$ is 1? 18. Of 9 d. 1 qr. is 3 d. 2 qr.

B. 1. $3 = \frac{1}{8}$ of what number?

1st Solution.—$3 = \frac{1}{8}$ of 8 times 3, or 24.

2d Solution.—If 3 is $\frac{1}{8}$ of some number, $\frac{8}{8}$ of the number must be 8 times 3, or 24. Hence, $3 = \frac{1}{8}$ of 24.

2. $9 = \frac{1}{7}$ of * ?
3. $7 = \frac{1}{8}$ of * ?
4. $8 = \frac{1}{4}$ of * ?
5. $\frac{5}{6} = \frac{1}{6}$ of * ?
6. $\frac{2}{3} = \frac{1}{9}$ of * ?
7. $\frac{3}{5} = \frac{1}{7}$ of * ?
8. $9\frac{1}{2} = \frac{1}{3}$ of * ?
9. $7\frac{2}{3} = \frac{1}{5}$ of * ?
10. 2 bu. 3 pk. = $\frac{1}{8}$ of * ?
11. 5 wk. 3 da. = $\frac{1}{8}$ of * ?

C. 1. $17 = \frac{3}{4}$ of what number ?

Solution.—If $17 = \frac{3}{4}$ of some number, $\frac{1}{4}$ of that number must be $\frac{1}{3}$ of 17, which is $5\frac{2}{3}$, and $\frac{4}{4}$ of the number must be 4 times $5\frac{2}{3}$, which is $22\frac{2}{3}$. Therefore, $17 = \frac{3}{4}$ of $22\frac{2}{3}$.

Note.—To prove this, see if $\frac{3}{4}$ of $22\frac{2}{3} = 17$.

2. $36 = \frac{4}{7}$ of * ?
3. $32 = \frac{4}{9}$ of * ?
4. $40 = \frac{5}{8}$ of * ?
5. $42 = \frac{6}{7}$ of * ?
6. $81 = \frac{9}{10}$ of * ?
7. $27 = \frac{7}{8}$ of * ?
8. $81 = \frac{8}{9}$ of * ?
9. $64 = \frac{8}{9}$ of * ?
10. $40 = \frac{5}{8}$ of * ?
11. $\frac{1}{9}$ of $45 = \frac{1}{7}$ of * ?
12. $3\frac{4}{5}$ times $10 = \frac{7}{8}$ of * ?
13. $9\frac{1}{7}$ times $7 = \frac{8}{9}$ of * ?
14. $5\frac{2}{3}$ times $9 = \frac{5}{6}$ of * ?
15. $8\frac{3}{4}$ times $8 = \frac{9}{10}$ of * ?
16. $8\frac{2}{3}$ times $9 = 1\frac{3}{7}$ times * ?
17. $9\frac{1}{3}$ times $6 = 2\frac{1}{4}$ times * ?
18. $8\frac{4}{5}$ times $10 = 1\frac{2}{7}$ times * ?
19. $3\frac{2}{3}$ times $9 = 3\frac{1}{2}$ times * ?
20. $5\frac{1}{2}$ times $8 = 1\frac{3}{4}$ times * ?
21. $8\frac{2}{3}$ times $6 = 2\frac{1}{3}$ times * ?

D. 1. If $\frac{3}{4}$ of a gallon of molasses cost 25 cents, what will 1 gallon cost ?

Solution.—If $\frac{3}{4}$ of a gallon cost 25 cents, $\frac{1}{4}$ of a gallon will cost $\frac{1}{3}$ of 25 cents, which is $8\frac{1}{3}$ cents; and $\frac{4}{4}$ of a gallon will cost 4 times $8\frac{1}{3}$ cents, which are $33\frac{1}{3}$ cents. Therefore, 1 gallon of molasses will cost $33\frac{1}{3}$ cents, if $\frac{3}{4}$ of a gallon cost 25 cents.

2. If $\frac{3}{8}$ of a yard of muslin cost 37 cents, what will 1 yard cost?

3. If $\frac{5}{7}$ of a yard of linen cost 53 cents, what will 1 yard cost?

4. If $\frac{3}{5}$ of a month's wages amount to 23 dollars, what will 1 month's wages amount to?

5. John is 17 years old, and his age is $\frac{3}{8}$ of his teacher's age. How old is his teacher?

6. Deborah says that she has 41 cents. "Then," said Lavinia, "you have only $\frac{7}{10}$ as many as I have." How many cents had Lavinia?

7. David told George that $\frac{5}{7}$ of his money would buy $5\frac{1}{3}$ pounds of raisins at 9 cents per pound. "Then," replied George, "you have 6 cents more than I have." How many cents did each of the boys have?

8. Seth's father gave him a half-dollar to buy a pound of tea with, saying to him, "$\frac{4}{5}$ of a pound of tea will cost 30 cents, and if you will tell me how much a pound will cost, you may have the money which will be left after paying for the tea." Seth answered correctly. What was his answer? How many cents would be left after paying for the tea?

9. A farmer gave $7\frac{2}{3}$ dozen of eggs at 9 cents per dozen for $\frac{7}{9}$ of a gallon of oil. What was a gallon of the oil worth?

10. A butcher received 35 shillings for $\frac{7}{8}$ of a hundred weight of beef. What would he receive for a hundred weight at the same price. What would he receive for $\frac{9}{10}$ of a hundred weight?

11. A schoolmaster being asked his age, replied, "$\frac{5}{8}$ of my life have been spent in teaching. I have taught in Boston 25 years, which is $\frac{5}{8}$ of all the time I have taught." What was his age?

12. If a yard of muslin costs 70 cents, and $\frac{3}{7}$ of a yard of muslin costs $\frac{5}{6}$ as much as a yard of cambric, what will a yard of cambric cost?

13. If a yard of linen costs 56 cents, and $\frac{6}{7}$ of a yard of linen costs $\frac{8}{9}$ as much as a yard of lawn, how much will a yard of lawn cost? How much will $\frac{7}{9}$ of a yard of lawn cost?

14. If Joseph can earn 54 cents in one day, and it takes William $\frac{3}{10}$ of a day to earn as much as Joseph can earn in $\frac{4}{9}$ of a day, how much can William earn in one day?

15. Mr. Battles gave 25 dollars for a cow, and $1\frac{2}{5}$ times what he gave for the cow is equal to $2\frac{1}{3}$ times what he gave for a heifer. What did he give for the heifer?

16. After Arthur had given $\frac{2}{9}$ of his money for a blank book, and $\frac{5}{9}$ of it for a grammar, he had 18 cents left. How many cents had he at first?

17. $\frac{2}{7}$ of Mr. Ball's farm is tillage, $\frac{4}{7}$ of it is pasturage and the rest, 3 acres, is orchard. How many acres does he own?

18. William and Henry were trying to puzzle each other about their ages. William told Henry that he had spent $\frac{1}{3}$ of his life in Philadelphia, $\frac{5}{8}$ of it in New York, and the rest in Boston, and that he had lived 5 years more in New York than in Boston. Henry found out his age. What was it?

19. Henry then said to William, "I have lived in Hartford, Providence, and Boston. I spent $\frac{1}{10}$ of my life in Hartford; I lived in Providence twice as long as in Hartford, and have lived in Boston 2 years more than 3 times as long as in Providence." William found out Henry's age. What was it?

LESSON LII.

A. 1. What is the effect of multiplying the numerator of the fraction $\frac{4}{19}$ by 3?

Ans.—Multiplying the numerator of the fraction $\frac{4}{19}$ by 3, gives $\frac{12}{19}$ for a result, which expresses 3 times as many parts, each of the same size as before, and is, therefore, 3 times as large. Hence, multiplying the numerator of $\frac{4}{19}$ by 3, multiplies the fraction by 3.

What is the effect of multiplying the numerator of—

2. $\frac{3}{11}$ by 2?
3. $\frac{4}{25}$ by 6?
4. $\frac{5}{7}$ by 3?
5. $\frac{8}{9}$ by 7?
6. $\frac{4}{7}$ by 5?
7. $\frac{3}{4}$ by 9?

8. What is the effect of dividing the numerator of the fraction $\frac{12}{13}$ by 6?

Ans.—Dividing the numerator of $\frac{12}{13}$ by 6, gives $\frac{2}{13}$ for a result, which expresses $\frac{1}{6}$ as many parts, each of the same size as before, and is therefore $\frac{1}{6}$ as large. Hence, dividing the numerator of $\frac{12}{13}$ by 6, divides the fraction by 6.

NOTE.—The first of the above solutions is equivalent to "3 times $\frac{4}{19} = \frac{12}{19}$, just as 3 times 4 apples = 12 apples;" and the second is equivalent to "$\frac{1}{6}$ of $\frac{12}{13} = \frac{2}{13}$, just as $\frac{1}{6}$ of 12 apples = 2 apples." They are necessary as a preparation for the exercises which follow.

What is the effect of dividing the numerator of—

9. $\frac{15}{17}$ by 5?
10. $\frac{10}{11}$ by 2?
11. $\frac{8}{9}$ by 4?
12. $\frac{9}{5}$ by 3?
13. $\frac{42}{53}$ by 7?
14. $\frac{72}{89}$ by 9?

Hence, *multiplying the numerator of a fraction multiplies the fraction, and dividing the numerator divides the fraction.*

B. **To the Teacher.**—Should the pupils find any difficulty in understanding the following exercises, illustrations should be given by dividing visible objects, such as apples, lines, &c., into various kinds of fractional parts.

From the nature of fractional parts, it follows that —

1st. The larger the number of fractional parts into which any unit is divided, or which it takes to equal that unit, the smaller each part will be.

2d. The smaller the number of fractional parts into which any unit is divided, or which it takes to equal that unit, the larger each part will be.

1. Which parts are larger in size, halves or fourths?

Ans. —Halves, because it takes a less number of them to equal a unit.

Which parts are larger in size —

2. Halves or thirds?
3. Fourths or eighths?
4. Thirds or ninths?
5. Halves or tenths?
6. Fourths or twelfths?
7. Fifths or twentieths?

C. 1. Each half equals how many sixths?

1st **Form of Answer.**—Each half equals ½ of 6 sixths, which is 3 sixths. Hence, each half equals 3 sixths.

2d **Form of Answer.**—Each half equals 3 sixths, for if a unit should be divided into 6 equal parts, ½ of the unit would contain 3 of them.

2. Each half equals how many fourths?
3. Each third equals how many ninths?
4. Each fourth equals how many twelfths?
5. Each fifth equals how many tenths?
6. Each sixth equals how many eighteenths?

D. From the foregoing exercises we may infer that—

1st. *Multiplying the number of fractional parts into which a unit is divided, or which it takes to equal a unit, divides each part.*

2d. *Dividing the number of parts into which a unit is divided, or which it takes to equal a unit, multiplies each part.*

1. What is the effect of multiplying the denominator of the fraction $\frac{3}{5}$ by 4?

Ans.—Multiplying the denominator of the fraction $\frac{3}{5}$ by 4, gives $\frac{3}{20}$ for a result, which expresses the same number of parts each $\frac{1}{4}$ as large as before. Hence, $\frac{3}{20} = \frac{1}{4}$ of $\frac{3}{5}$, or multiplying the denominator of $\frac{3}{5}$ by 4, has divided the fraction by 4.

What is the effect of multiplying the denominator of—

2. $\frac{2}{3}$ by 5?
3. $\frac{3}{5}$ by 6?
4. $\frac{2}{3}$ by 2?
5. $\frac{4}{7}$ by 3?
6. $\frac{5}{8}$ by 3?
7. $\frac{3}{4}$ by 4?

What is the effect of dividing the denominator of $\frac{5}{12}$ by 2?

Ans.—Dividing the denominator of the fraction $\frac{5}{12}$ by 2, gives $\frac{5}{6}$ for a result, which expresses the same number of parts each twice as large as before. Hence, $\frac{5}{6}$ = two times $\frac{5}{12}$, or the fraction $\frac{5}{12}$ has been divided by 2.

What is the effect of dividing the denominator —

9. Of $\frac{4}{9}$ by 3? 11. Of $\frac{7}{24}$ by 8? 13. Of $\frac{4}{21}$ by 7?
10. Of $\frac{5}{6}$ by 2? 12. Of $\frac{3}{25}$ by 5? 14. Of $\frac{11}{18}$ by 9?

E. The numerator and denominator are called terms of the fraction.

1. What is the effect of multiplying both terms of $\frac{2}{3}$ by 6?

Ans.—Multiplying both terms of the fraction $\frac{2}{3}$ by 6, gives $\frac{12}{18}$ for a result, which expresses 6 times as many parts, each $\frac{1}{6}$ as large as before. Hence, the value of the fraction is not altered, or $\frac{2}{3} = \frac{12}{18}$.

What is the effect of multiplying both terms of —

2. $\frac{1}{2}$ by 3? 5. $\frac{9}{10}$ by 3? 8. $\frac{4}{5}$ by 8?
3. $\frac{3}{4}$ by 2? 6. $\frac{3}{8}$ by 4? 9. $\frac{2}{11}$ by 5?
4. $\frac{5}{7}$ by 4? 7. $\frac{2}{9}$ by 7? 10. $\frac{8}{9}$ by 6?

11. What is the effect of dividing both terms of $\frac{15}{25}$ by 5.

Ans.—Dividing both terms of the fraction $\frac{15}{25}$ by 5, gives $\frac{3}{5}$ for a result, which expresses $\frac{1}{5}$ as many parts each 5 times as large as before. Hence, the value of the fraction is not altered, or $\frac{15}{25} = \frac{3}{5}$.

What is the effect of dividing both terms of —

12. $\frac{9}{12}$ by 3? 15. $\frac{3}{12}$ by 3? 18. $\frac{14}{49}$ by 7?
13. $\frac{4}{8}$ by 4? 16. $\frac{9}{15}$ by 3? 19. $\frac{35}{45}$ by 5?
14. $\frac{10}{15}$ by 5? 17. $\frac{14}{21}$ by 7? 20. $\frac{81}{99}$ by 9?

F. **From the foregoing, it appears that —**

1. *Multiplying the numerator multiplies the fraction, by multiplying the number of parts considered, without affecting their size.*

2. *Dividing the numerator divides the fraction, by dividing the number of parts considered, without affecting their size.*

3. *Multiplying the denominator divides the fraction, by dividing each part, without affecting the number of parts considered.*

4. *Dividing the denominator multiplies the fraction, by multiplying each part, without affecting the number of parts considered.*

5. *A fraction may be multiplied either by multiplying the numerator or by dividing the denominator.*

6. *A fraction may be divided either by dividing the numerator or by multiplying the denominator.*

7. *Multiplying both numerator and denominator of a fraction by the same number both multiplies and divides the fraction by that number, and, therefore, does not alter its value.*

8. *Dividing both numerator and denominator of a fraction by the same number, both divides and multiplies the fraction by that number, and, therefore, does not alter its value.*

LESSON LIII.

A. A fraction is said to be in its LOWEST TERMS when its numerator and denominator are the smallest entire numbers which will express its value.

When a fraction is in its Lowest Terms, there is no entire number greater than one which will divide both numerator and denominator without a remainder.

Hence, *a fraction may be reduced to its lowest terms by dividing both numerator and denominator by the same numbers.*

1. Reduce $\frac{18}{24}$ to its lowest terms.

Solution. — Both terms of $\frac{18}{24}$ can be divided by 6. Dividing them, gives $\frac{3}{4}$ for a result, which expresses $\frac{1}{6}$ as many parts, each 6 times as large as before. Hence, $\frac{18}{24} = \frac{3}{4}$, and as $\frac{3}{4}$ admits of no further reduction, it is the fraction required.

The number by which we divide in reducing fractions to their lowest terms, are said to be CANCELED. Thus, in the first solution, we canceled the factor 6; in the second, we canceled the factor 2, and then the factor 3.

Note. — The pupil should not only master the explanation, but should also learn to give the results without the explanation. Let him also observe that a fraction can always be reduced to its lowest terms by dividing both terms by their greatest common divisor.

Reduce each of the following fractions to its lowest terms:—

2. $\frac{8}{12}$.
3. $\frac{9}{15}$.
4. $\frac{8}{24}$.
5. $\frac{12}{15}$.
6. $\frac{16}{20}$.
7. $\frac{36}{64}$.
8. $\frac{42}{56}$.
9. $\frac{28}{35}$.
10. $\frac{18}{27}$.
11. $\frac{18}{20}$.
12. $\frac{48}{72}$.
13. $\frac{54}{72}$.

B. A fraction is sometimes expressed by the factors of its numerator and denominator.

Example.—The fraction $\frac{4 \times 9}{15 \times 8}$, which may be read, "The fraction having 4 times 9 for its numerator, and 15 times 8 for its denominator," or, "The fraction 4 times 9, divided by 15 times 8."

Such fractions should be reduced to their lowest terms before multiplying their factors together.

Reduce $\frac{4 \times 9}{15 \times 8}$ to its lowest terms.

Solution.—Canceling 4 from the factor 4 of the numerator and 8 of the denominator, gives 1 in place of the former, and 2 in place of the latter.* Cancelling 3 from the factor 9 in the numerator, and 15 of the denominator, gives 3 in place of the former, and 5 in place of the latter.† As no further division can be made, we multiply the remaining factors together, which gives $\frac{4 \times 9}{15 \times 8} = \frac{1 \times 3}{5 \times 2} = \frac{3}{10}.$

In writing out the work, it is customary to draw a line through the numbers from which factors have been canceled, and to write the quotients above the dividends of the numerator, and below those of the denominator.

Reduce each of the following to its lowest terms:

2. $\frac{6 \times 4}{9 \times 8}$

3. $\frac{10 \times 9}{21 \times 5}$

4. $\frac{12 \times 7}{35 \times 18}$

5. $\frac{16 \times 18}{45 \times 24}$

6. $\frac{28 \times 36}{24 \times 14}$

7. $\frac{49 \times 25}{35 \times 35}$

8. $\frac{2 \times 3 \times 4}{6 \times 5 \times 8}$

9. $\frac{4 \times 7 \times 9}{7 \times 6 \times 8}$

10. $\frac{3 \times 4 \times 6}{6 \times 8 \times 3}$

11. $\frac{4 \times 9 \times 21}{14 \times 6 \times 3}$

12. $\frac{15 \times 7 \times 13}{13 \times 35 \times 9}$

13 $\frac{24 \times 18 \times 25}{36 \times 45 \times 56}.$

LESSON LIV.

A. 1. How can ½ of a fraction be found?

Ans.—½ of a fraction can be found by dividing its numerator by 2; or by multiplying its denominator by 2. For, dividing the numerator by 2, will give for a result a fraction expressing ½ as many parts, each of the same size as those of the given fraction;

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* This makes the fraction express ¼ as many parts, each 4 times as large as before, and hence does not alter its value.

† This makes the fraction express ⅓ as many parts, each 3 times as large as before, and therefore does not alter its value.

or, dividing the denominator, will give for a result a fraction expressing the same number of parts, each $\frac{1}{2}$ as large as those of the given fraction.

Note.—The statement of the reasons should not be omitted till it is *certain* that the pupil fully understands them.

State the method of finding —

2. $\frac{1}{3}$ of a fraction.
3. $\frac{1}{7}$ of a fraction.
4. $\frac{1}{9}$ of a fraction.
5. $\frac{1}{12}$ of a fraction.
6. $\frac{1}{8}$ of a fraction.
7. $\frac{1}{10}$ of a fraction.

B. 1. What is the effect of multiplying the numerator of a fraction by 3, and the denominator by 4?

Ans.—Multiplying the numerator of a fraction by 3, and the denominator by 4, gives for a result $\frac{3}{4}$ of the original fraction, for it gives 3 times as many parts, each $\frac{1}{4}$ as large as before.

What is the effect of multiplying the numerator of a fraction —

2. By 4, and the denominator by 7?
3. By 8, and the denominator by 3?
4. By 11, and the denominator by 6?
5. By 12, and the denominator by 10?
6. By 24, and the denominator by 17?

C. 1. How can $\frac{5}{6}$ of a fraction be found?

1st **Method.**—$\frac{5}{6}$ of a fraction can be found by getting $\frac{5}{6}$ of the numerator for a new numerator, without altering the denominator.

2d **Method.** — $\frac{5}{6}$ of a fraction can be found by multiplying the numerator by 5 and the denominator by 6.

Note. — The pupil should observe that the first method gives $\frac{5}{6}$ as many parts of the same size as before, and that the second gives 5 times as many parts, each $\frac{1}{6}$ as large as before. He should observe, further, that the first method is identical with that of Lesson L.

Explain the methods of finding —

2. $\frac{7}{8}$ of a fraction.
3. $\frac{6}{7}$ of a fraction.
4. $\frac{3}{11}$ of a fraction.
5. $\frac{4}{13}$ of a fraction.
6. $\frac{2}{3}$ of a fraction.
7. $\frac{13}{14}$ of a fraction.
8. $\frac{5}{12}$ of a fraction.
9. $\frac{5}{9}$ of a fraction

D. 1. What is $\frac{5}{6}$ of $\frac{9}{10}$?

Ans.— $\frac{5}{6}$ of $\frac{9}{10}$ $= \dfrac{\overset{1}{\cancel{5}} \times \overset{3}{\cancel{9}}}{\underset{2}{\cancel{6}} \times \underset{2}{\cancel{10}}} = \frac{3}{4}$.

2. $\frac{7}{8}$ of $\frac{3}{5}$?
3. $\frac{5}{8}$ of $\frac{4}{15}$?
4. $\frac{4}{13}$ of $\frac{5}{8}$?
5. $\frac{3}{8}$ of $\frac{7}{8}$?
6. $\frac{3}{4}$ of $\frac{2}{3}$?
7. $\frac{24}{25}$ of $\frac{35}{36}$?
8. $\frac{36}{49}$ of $\frac{14}{32}$?
9. $\frac{2}{3}$ of $\frac{3}{4}$?
10. $\frac{8}{9}$ of $\frac{15}{16}$?
11. $\frac{9}{16}$ of $\frac{4}{5}$?
12. $\frac{7}{8}$ of $\frac{20}{21}$?
13. $\frac{4}{11}$ of $\frac{9}{10}$?

E. 1. What is the product of $\frac{5}{8}$ times $\frac{12}{25}$?

Solution.— $\frac{5}{8}$ times $\frac{12}{25} = \frac{5}{8}$ of $\frac{12}{25} = \dfrac{\cancel{5} \times \overset{3}{\cancel{12}}}{\underset{2}{\cancel{8}} \times \underset{5}{\cancel{25}}} = \frac{3}{10}$.

2. What is the product of $\frac{9}{14} \times \frac{28}{33}$?

1st Solution.— $\frac{9}{14} \times \frac{28}{33}$, or $\frac{9}{14}$ times $\frac{28}{33} = \frac{9}{14}$ of $\frac{28}{33} = \dfrac{9 \times 28}{14 \times 33} = \frac{6}{11}$.

2d. Solution. — $\frac{9}{14} \times \frac{28}{33}$, or $\frac{9}{14}$ multiplied by $\frac{28}{33}$, $= \frac{28}{33}$ of $\frac{9}{14} = \dfrac{28 \times 9}{33 \times 14} = \frac{6}{11}$.

Note.—The slight difference between the first and second solution results from the different reading of the sign of multiplication. We recommend the first as being the most simple.

3. $\frac{3}{5} \times \frac{11}{12}$? 6. $\frac{20}{27} \times \frac{63}{80}$? 9. $\frac{5}{6} \times \frac{3}{8} \times \frac{16}{15}$?
4. $\frac{11}{12} \times \frac{3}{5}$? 7. $\frac{32}{81} \times \frac{54}{64}$? 10. $\frac{3}{4} \times \frac{5}{6} \times \frac{8}{15}$?
5. $\frac{6}{7} \times \frac{7}{12}$? 8. $\frac{42}{55} \times \frac{33}{28}$? 11. $\frac{4}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{16}$?

12. $4\frac{1}{2} \times 2\frac{5}{6}$?

Solution.—$4\frac{1}{2} \times 2\frac{5}{6} = \frac{9}{2} \times \frac{17}{6} = \frac{51}{4} = 12\frac{3}{4}$.

13. $2\frac{2}{3} \times 4\frac{1}{2}$? 15. $2\frac{1}{2} \times 3\frac{1}{3}$? 17. $1\frac{3}{5} \times 1\frac{1}{3}$?
14. $5\frac{3}{4} \times 1\frac{1}{7}$? 16. $4\frac{2}{7} \times 1\frac{3}{4}$? 18. $3\frac{1}{3} \times 7\frac{1}{2}$?

G. 1. $\frac{3}{4} = \frac{6}{7}$ of what number?

Solution.—If $\frac{3}{4} = \frac{6}{7}$ of some number, $\frac{1}{7}$ of that number must be $\frac{1}{6}$ of $\frac{3}{4}$, which is $\frac{3}{6 \times 4} = \frac{1}{8}$; and $\frac{7}{7}$ of the number must be 7 times $\frac{1}{8}$, which are $\frac{7}{8}$. Hence, $\frac{3}{4} = \frac{6}{7}$ of $\frac{7}{8}$.

2. $\frac{2}{3} = \frac{4}{7}$ of * ? 6. $\frac{5}{8} = \frac{50}{12}$ of * ?
3. $\frac{24}{25} = \frac{8}{15}$ of * ? 7. $3\frac{1}{3} = 2\frac{1}{2}$ times * ?
4. $\frac{9}{10} = 1\frac{1}{3}$ times * ? 8. $2\frac{1}{4} = 2\frac{2}{3}$ times * ?
5. $\frac{7}{8} = 2\frac{1}{3}$ times * ? 9. $4\frac{3}{4} = \frac{5}{6}$ of * ?

H. 1. How much will $\frac{3}{4}$ of a yard of cloth cost at $\frac{5}{8}$ of a dollar per yard?

2. How much will $\frac{2}{3}$ of a quart of filberts cost at $\frac{4}{5}$ of a dime per quart?

3. George gathered $\frac{5}{8}$ of a bushel of cranberries, and sold $\frac{2}{3}$ of what he gathered. What part of a bushel did he sell?

4. Rufus earned $\frac{7}{8}$ of a dollar, and then spent $\frac{2}{3}$ of what he had earned. What part of a dollar did he spend?

5. If 1 pound of tea is worth $\frac{4}{5}$ of a dollar, what is $\frac{5}{7}$ of a pou worth?

6. If a man can hoe $\frac{8}{9}$ of an acre of corn in 1 day, what p of an acre can he hoe in $\frac{3}{4}$ of a day?

7. If 5 pounds of soap cost $\frac{1}{3}$ of a dollar, what will one pou cost?

8. If 4 oranges cost $\frac{1}{3}$ of a dollar, what will 1 orange co What will 3 cost?

9. If 6 pine apples cost $\frac{3}{4}$ of a dollar, what will 5 cost?

10. If 5 lb. of coffee cost $\frac{3}{7}$ of a dollar, what will 10 lb. cost

11. If $\frac{2}{3}$ of a yard of silk velvet cost $5\frac{1}{4}$ dollars, what wil yard cost?

SOLUTION.—If $\frac{2}{3}$ of a yard of silk velvet cost $5\frac{1}{4}$ dollars, $\frac{1}{3}$ of a y will cost $\frac{1}{2}$ of $5\frac{1}{4}$ dollars. $\frac{1}{2}$ of $5\frac{1}{4} = \frac{1}{2}$ of $4 + \frac{1}{2}$ of $1\frac{1}{4}$, or of $\frac{5}{4}$, whic $2\frac{5}{8}$. If $\frac{1}{3}$ of a yard cost $2\frac{5}{8}$ dollars, $\frac{3}{3}$ of a yard will cost 3 times dollars, which are $7\frac{7}{8}$ dollars.

12. If $\frac{3}{8}$ of an acre cost $30\frac{1}{2}$ dollars, what will 1 acre cost?

13. If $\frac{7}{9}$ of a cask of oil is worth $64\frac{2}{5}$ dollars, what is the e worth?

14. If $2\frac{1}{2}$ cords of wood are worth \$$18\frac{3}{4}$, what is 1 cord wor

15. If a wood-cutter can cut $5\frac{3}{5}$ acres of wood in $2\frac{1}{3}$ days, h much can he cut in 1 day?

16. If Rufus can shell $2\frac{3}{5}$ bushels of corn in 1 hour, how m bushels can he shell in $2\frac{2}{3}$ hours?

17. If Rufus can shell $7\frac{1}{5}$ bushels of corn in $2\frac{3}{5}$ hours, how m bushels can he shell in 1 hour?

18. If Albert can walk $18\frac{1}{3}$ miles in 4 hours, how far can walk in 1 hour? How far in $2\frac{1}{2}$ hours?

19. If Albert can walk $7\frac{1}{3}$ miles in $2\frac{1}{4}$ hours, how far can walk in 1 hour? How far in 4 hours?

20. When $4\frac{1}{2}$ bushels of corn can be bought for $\$2\frac{3}{5}$, how many bushels can be bought for 1 dollar? How many for $\frac{9}{10}$ of a dollar?

21. When $1\frac{1}{10}$ bushels of corn can be bought for $\frac{9}{10}$ of a dollar, how many bushels can be bought for 1 dollar? How many for $\$2\frac{3}{5}$?

22. If $\frac{3}{4}$ of a yard of linen is given in exchange for $\frac{3}{5}$ of a yard of silk worth $\frac{5}{6}$ of a dollar per yard, what ought the linen to be worth per yard?

23. If $\frac{3}{5}$ of a yard of silk is given in exchange for $\frac{3}{4}$ of a yard of linen worth $\frac{2}{3}$ of a dollar per yard, what ought the silk to be worth per yard?

24. What part of 1 rod is 4 yd. 2 ft. $1\frac{5}{7}$ in.?

SOLUTION.—Since 1 in. $=\frac{1}{12}$ of a foot, $1\frac{5}{7}$ in. or $\frac{12}{7}$ of an inch must equal $\frac{12}{7}$ of $\frac{1}{12}$ of a ft. $=\frac{1}{7}$ ft., to which, adding the 2 ft., gives $2\frac{1}{7}$ ft., or $\frac{15}{7}$ ft. Since 1 ft. $=\frac{1}{3}$ of a yd., $\frac{15}{7}$ of a ft. must equal $\frac{15}{7}$ of $\frac{1}{3}$ of a yd. $=\frac{5}{7}$ yd., to which, adding the 4 yd., gives $4\frac{5}{7}$ yd. $=\frac{33}{7}$ yd. Since 1 yd. $=\frac{2}{11}$ of a rd. $\frac{33}{7}$ of a yd. must equal $\frac{33}{7}$ of $\frac{2}{11}$ rd. $=\frac{6}{7}$ rd. Hence, 4 yd. 2 ft. $1\frac{5}{7}$ in. $=\frac{6}{7}$ of a rod.

Prove by Solution to 13th problem, page 131.

25. What part of 1 bu. is 3 pk. 1 qt. $1\frac{1}{3}$ pt.?

26. Of 1 gal. is 2 qt. 1 pt. $2\frac{6}{7}$ gi.?

27. Of 1 lb. is 8 oz. $14\frac{2}{3}$ dr.?

28. Of 1 wk. is 5 da. 10 h. 40 m.?

29. Of 1 £. is 2 s. 10 d. $1\frac{1}{7}$ qr.

30. Of 1 ℔. is 9 ℥. 4 ʒ. 2 ℈. 8 gr.?

31. Of 1 lb is 4 oz. 5 dwt. $17\frac{1}{7}$ gr.?

32. Of 1 T. is 2 cwt. 0 qr. 22 lb. 3 oz. $8\frac{3}{5}$ oz.

LESSON LV.

A. 1. $1 =$ how many times $\frac{1}{5}$?

Solution.—$1 = \frac{5}{5}$, and $\frac{5}{5} = 5$ times $\frac{1}{5}$. Hence, $1 = 5$ times $\frac{1}{5}$.

2. What is the quotient of $1 \div \frac{1}{7}$?

Solution.—$1 = \frac{7}{7}$, and $\frac{7}{7} \div \frac{1}{7} = 7 \div 1 = 7$. Hence, $1 \div \frac{1}{7} = 7$.

3. $1 =$ * times $\frac{1}{2}$?
4. $1 =$ * times $\frac{1}{9}$?
5. $1 =$ * times $\frac{1}{12}$?
6. $1 =$ * times $\frac{1}{20}$?
7. $1 \div \frac{1}{4}$?
8. $1 \div \frac{1}{7}$?
9. $1 \div \frac{1}{8}$?
10. $1 \div \frac{1}{6}$?

Inference.—Since $1 = 2$ times $\frac{1}{2}$, $= 3$ times $\frac{1}{3}$, &c., it follows that there will be 2 times as many halves, 3 times as many thirds, &c., as there are times 1 in any number.

B. 1. $5 =$ * times $\frac{1}{3}$?

Solution.—Since 5 contains 1, 5 times, it must contain $\frac{1}{3}$, 3 times 5 times, or 15 times. Hence, $5 = 15$ times $\frac{1}{3}$.

2. What is the quotient of $8 \div \frac{1}{3}$?

Solution.—Since the quotient of $8 \div 1 = 8$, the quotient of $8 \div \frac{1}{3}$ must equal 3 times 8 or 24. Hence, $8 \div \frac{1}{3} = 24$.

3. $7 =$ * times $\frac{1}{4}$?
4. $8 =$ * times $\frac{1}{6}$?
5. $5 =$ * times $\frac{1}{10}$?
6. $9 =$ * times $\frac{1}{7}$?
7. $4 =$ * times $\frac{1}{3}$?
8. $6 =$ * times $\frac{1}{2}$?
9. $8 \div \frac{1}{7}$?
10. $2 \div \frac{1}{9}$?
11. $10 \div \frac{1}{4}$?
12. $4 \div \frac{1}{3}$?
13. $12 \div \frac{1}{5}$?
14. $9 \div \frac{1}{8}$?

C. From all the preceding exercises, it must be obvious that the quotient of a number divided by 1 equals that number. Thus $3 \div 1 = 3$; $7 \div 1 = 7$, &c. So $\frac{2}{3} \div 1 = \frac{2}{3}$; $\frac{5}{8} \div 1 = \frac{5}{8}$, &c.

1. $\frac{3}{5}$ = how many times $\frac{1}{4}$?

Solution. — Since $\frac{3}{5}$ contains 1, $\frac{3}{5}$ times, it must contain $\frac{1}{4}$, 4 times $\frac{3}{5}$ times, which are $\frac{12}{5}$ times $= 2\frac{2}{5}$ times. Hence, $\frac{3}{5} = 2\frac{2}{5}$ times $\frac{1}{4}$.

2. What is the quotient of $\frac{5}{8} \div \frac{1}{7}$?

Solution.—Since the quotient of $\frac{5}{8}$ divided by $1 = \frac{5}{8}$, the quotient of $\frac{5}{8}$ divided by $\frac{1}{7}$ must equal 7 times $\frac{5}{8}$, which are $\frac{35}{8} = 4\frac{3}{8}$. Hence, $\frac{5}{8} \div \frac{1}{7} = 4\frac{3}{8}$.

3. $\frac{4}{7} = *$ times $\frac{1}{8}$?
4. $\frac{3}{11} = *$ times $\frac{1}{5}$?
5. $\frac{3}{5} = *$ times $\frac{1}{7}$?
6. $\frac{5}{9} = *$ times $\frac{1}{8}$?
7. $\frac{3}{8} = *$ times $\frac{1}{4}$?
8. $\frac{2}{3} \div \frac{1}{4}$?
9. $\frac{5}{7} \div \frac{1}{6}$?
10. $\frac{3}{5} \div \frac{1}{6}$?
11. $\frac{8}{9} \div \frac{1}{10}$?
12. $\frac{4}{11} \div \frac{1}{3}$?

D. From the nature of division, it is obvious that, *while the dividend remains the same*, the larger the divisor is, the smaller will be the quotient, and the smaller the divisor is, the larger will be the quotient.

Thus the quotient of 8 divided by 2 is 4, which is $\frac{1}{2}$ of the quotient of 8 divided by 1; the quotient of $15 \div 3$ is 5, which is $\frac{1}{3}$ of the quotient of 15 divided by 1. So the quotient of a number divided by $\frac{4}{5}$ must be $\frac{1}{4}$ of its quotient divided by $\frac{1}{5}$; the quotient of a number divided by $\frac{3}{7}$ must be $\frac{1}{3}$ of its quotient divided by $\frac{1}{7}$, &c., &c.

1. $8 = * \frac{3}{5}$?

Solution.—Since 8 contains $\frac{1}{5}$, 5 times 8 times, it must contain $\frac{3}{5}$ $\frac{1}{3}$ of 5 times 8 times, or $\frac{5}{3}$ of 8 times, which are $13\frac{1}{3}$ times. Hence, $8 = 13\frac{1}{3}$ times $\frac{3}{5}$.

2. What is the quotient of 4 $\div \frac{3}{7}$?

SOLUTION.—Since the quotient of 4 divided by $\frac{1}{7} = 7$ times 4, quotient of 4 divided by $\frac{3}{7}$ must equal $\frac{1}{3}$ of 7 times 4, or $\frac{7}{3}$ of 4, w is $9\frac{1}{3}$. Hence, $4 \div \frac{3}{7} = 9\frac{1}{3}$.

3. 7 = * times $\frac{2}{3}$?
4. 5 = * times $\frac{4}{5}$?
5. 8 = * times $\frac{3}{4}$?
6. 1 = * times $\frac{2}{9}$?
7. 1 = * times $\frac{7}{8}$?
8. 4 = * times $\frac{5}{12}$?
9. $8 \div \frac{5}{11}$?
10. $2 \div \frac{7}{12}$?
11. $6 \div \frac{2}{9}$?
12. $1 \div \frac{3}{8}$?
13. $1 \div \frac{2}{5}$?
14. $9 \div \frac{4}{7}$?

E. 1. $\frac{2}{5}$ = how many times $\frac{3}{7}$?

SOLUTION.—Since $\frac{2}{5}$ contains $\frac{1}{7}$, 7 times $\frac{2}{5}$ times, it must contai $\frac{1}{3}$ of 7 times $\frac{2}{5}$ times or $\frac{7}{3}$ of $\frac{2}{5}$ times $= \frac{14}{15}$ times. Hence, $\frac{2}{5} =$ times $\frac{3}{7}$.

2. What is the quotient of $\frac{3}{4} \div \frac{6}{7}$?

SOLUTION.—Since the quotient of $\frac{3}{4}$ divided by $\frac{1}{7} = 7$ times $\frac{3}{4}$, quotient of $\frac{3}{4}$ divided by $\frac{6}{7}$ must equal $\frac{1}{6}$ of 7 times $\frac{3}{4}$, or $\frac{7}{6}$ of $\frac{3}{4}$, wh by canceling the factor 3, equals $\frac{7}{8}$. Hence, $\frac{3}{4} \div \frac{6}{7} = \frac{7}{8}$.

3. $\frac{3}{5}$ = * times $\frac{9}{10}$?
4. $\frac{1}{3}$ = * times $\frac{6}{5}$?
5. $\frac{2}{7}$ = * times $\frac{4}{11}$?
6. $\frac{5}{9}$ = * times $\frac{3}{8}$?
7. $\frac{2}{7}$ = * times $\frac{5}{11}$?
8. $\frac{8}{9} \div \frac{4}{5}$?
9. $9\frac{1}{7} \div \frac{3}{8}$?
10. $\frac{9}{10} \div \frac{3}{8}$?
11. $\frac{5}{8} \div \frac{2}{7}$?
12. $\frac{8}{9} \div \frac{8}{11}$?

F. The preceding exercises and solutions make it evident the quotient of a number divided by $\frac{2}{3} = \frac{1}{2}$ of 3 times the n ber $= \frac{3}{2}$ of the number; the quotient of a number dividec $\frac{4}{9} = \frac{1}{4}$ of 9 times the number $= \frac{9}{4}$ of the number, and gener that —

The quotient of a number divided by a fraction equals the product of that number multiplied by the fraction inverted.

1. What is the quotient of $\frac{4}{9} \div 2\frac{2}{3}$?

Ans.—$\frac{4}{9} \div 2\frac{2}{3} = \frac{4}{9} \div \frac{8}{3} = \frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$.

2. $\frac{4}{7} \div 1\frac{1}{3}$? 5. $2\frac{1}{4} \div 4\frac{1}{2}$? 8. $8\frac{1}{3} \div 4\frac{1}{6}$?

3. $\frac{3}{5} \div 7\frac{1}{2}$? 6. $\frac{15}{16} \div 3\frac{1}{4}$? 9. $\frac{9}{16} \div \frac{5}{6}$?

4. $5\frac{1}{4} \div 4\frac{2}{3}$? 7. $9\frac{3}{5} \div 1\frac{5}{7}$? 10. $8\frac{1}{4} \div 5\frac{3}{8}$?

G. Examples in division of fractions sometimes appear in the form of fractions. They are then called COMPLEX FRACTIONS.

ILLUSTRATION. — $\frac{4\frac{2}{3}}{3\frac{1}{5}}$, which equals $4\frac{2}{3} \div 3\frac{1}{5}$.

A complex fraction, then, has a fraction in one or both its terms. They may be explained after the following model: —

$\frac{4\frac{2}{3}}{3\frac{1}{5}}$ expresses the value of $4\frac{2}{3}$ equal parts of such kind that $3\frac{1}{5}$ of them will equal a unit.

Complex fractions may be reduced to simple ones by merely performing the indicated division.

Thus: $\frac{4\frac{2}{3}}{3\frac{1}{5}} = 4\frac{2}{3} \div 3\frac{1}{5} = \frac{14}{3} \div \frac{16}{5} = \frac{14 \times 5}{3 \times 16} = \frac{35}{24} = 1\frac{11}{24}$.

Reduce each of the following to simple fractions:

1. $\frac{1\frac{2}{3}}{2\frac{1}{2}}$. 3. $\frac{8\frac{1}{2}}{4\frac{2}{3}}$. 5. $\frac{4\frac{1}{5}}{6\frac{3}{16}}$.

2. $\frac{3\frac{3}{4}}{2\frac{1}{7}}$. 4. $\frac{5\frac{1}{7}}{8\frac{3}{4}}$. 6. $\frac{3\frac{8}{9}}{1\frac{1}{6}}$.

H. 1. How many melons at $\frac{3}{4}$ of a dime each can be bought for 5 dimes?

SOLUTION.—Since 1 melon can be bought for $\frac{3}{4}$ of a dime, as many melons can be bought for 5 dimes as there are times $\frac{3}{4}$ in 5, which are $\frac{4}{3}$ of 5 times or $\frac{20}{3}$ times $= 6\frac{2}{3}$ times. Hence, $6\frac{2}{3}$ melons at $\frac{3}{4}$ of a dime a-piece can be bought for 5 dimes.

2. How many bushels of corn at $\frac{5}{8}$ of a dollar per bushel can be bought for 7 dollars?

3. How many hours will it take a scholar who learns $\frac{8}{7}$ of a page per hour to learn 8 pages?

4. When tea is worth $\frac{4}{9}$ of a dollar per lb., how many pounds can be bought for $5.

5. If a man can walk $\frac{5}{12}$ of a furlong in 1 minute, how many minutes will it take him to walk $\frac{7}{8}$ of a furlong?

6. If a man can gather $\frac{2}{9}$ of the apples on a certain tree in 1 hour, how many hours will it take him to gather $\frac{7}{12}$ of them?

7. Edward divided $\frac{2}{7}$ of a rood of land into flower-beds, each containing $\frac{1}{14}$ of a rood. How many beds did he make?

8. A man who had $5, gave $\frac{1}{3}$ of his money for grass seed at $$2\frac{1}{2}$ per bushel. How many bushels did he buy?

9. How many pounds of pearlash at $\frac{9}{10}$ of a dime per pound can be bought for $\frac{3}{5}$ of a pound of chocolate at $3\frac{3}{4}$ dimes per lb.?

10. If Josephine can learn $\frac{7}{9}$ of a lesson in an hour, how many hours will it take her to learn 1 lesson?

11. Albert has a cord 28 feet long, which he wishes to cut into pieces each $2\frac{4}{5}$ feet long. How many pieces will it make?

12. A man who had but $9, invested $\frac{3}{4}$ of his money in cloth at $1\frac{1}{2}$ dollars per yard, and the rest of it in cloth at $1\frac{1}{5}$ dollars per yard. How many yards of each kind did he buy?

13. When a bushel of potatoes can be bought for $\frac{3}{8}$ of a dollar, how many bushels of potatoes can be bought for 9 bushels of corn at $\frac{1}{2}$ of a dollar per bushel?

14. How many bottles, each holding $\frac{4}{5}$ of a quart, can be filled from $\frac{7}{8}$ of a gallon of wine?

15. A farmer has $2\frac{3}{7}$ tons of hay in one stack, and $3\frac{6}{7}$ tons in another. He carries it to market in loads each weighing $1\frac{3}{8}$ tons. How many loads will both stacks make?

16. How many tiles $\frac{3}{4}$ of a foot long and $\frac{1}{2}$ of a foot wide will it take to cover 18 sq. ft. of surface?

17. How many square yards in a floor 12 feet long and 9 feet wide, and how many yards of carpeting $\frac{3}{4}$ of a yard wide, will it take to cover it?

18. When $1 is received for $\frac{2}{3}$ of a sq. ft. of land, how many dollars will be received for a strip 16 feet long, and $\frac{5}{12}$ of a foot wide?

LESSON LVI.

A. 1. $\frac{3}{4}$ = how many twelfths?

Solution.—Since $1 = \frac{12}{12}$, $\frac{3}{4}$ of 1 must equal $\frac{3}{4}$ of $\frac{12}{12}$, which are $\frac{9}{12}$. Hence, $\frac{3}{4} = \frac{9}{12}$.

In a similar manner reduce —

2.	$\frac{2}{3}$ to sixths.	9.	$\frac{3}{4}$ and $\frac{1}{3}$ to twelfths.
3.	$\frac{1}{2}$ to eighths.	10.	$\frac{5}{6}$ and $\frac{4}{9}$ to thirty-sixths.
4.	$\frac{4}{5}$ to tenths.	11.	$\frac{3}{4}$ and $\frac{3}{10}$ to fortieths.
5.	$\frac{3}{4}$ to twentieths.	12.	$\frac{7}{12}$ and $\frac{3}{8}$ to twenty-fourths.
6.	$\frac{5}{9}$ to forty-fifths.	13.	$\frac{9}{10}$ and $\frac{7}{30}$ to sixtieths.
7.	$\frac{4}{7}$ to twenty-firsts.	14.	$\frac{3}{8}$ and $\frac{4}{7}$ to fifty-sixths.
8.	$\frac{11}{12}$ to thirty-sixths.	15.	$\frac{4}{5}$, $\frac{1}{7}$ and $\frac{3}{10}$ to seventieths.

Let the pupil now solve the above questions by the following form:—

Solution to problem 1st.—Since the required denominator, 12, is 3 times the given denominator, 4, we multiply both terms of $\frac{3}{4}$ by 3, which gives $\frac{3}{4} = \frac{9}{12}$.

B. 1. Fractions have a COMMON DENOMINATOR when they have the same denominator.

Illustration.—$\frac{3}{4}$ and $\frac{1}{3}$ do not have a common denominator, but $\frac{5}{9}$, $\frac{8}{9}$, and $\frac{2}{9}$ have the common denominator 9.

2. Fractions having different denominators can be reduced to a common denominator, *i. e.*, to equivalent fractions having a common denominator. This is illustrated in the last 7 examples under A.

3. In reducing fractions to a common denominator —

1st. *Select a convenient number for the common denominator.*

2d. *Reduce the given fractions by the method explained in A.*

4. It will usually be most convenient to select the least common multiple of the denominators of the given fractions for a common denominator.

C. 1. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to a common denominator.

Partial Solution.—We first find the least common multiple of the given denominators 6, 8, and 12. It is 24, which we therefore select for the common denominator. The problem is now equivalent to the following: "Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to twenty-fourths," and may be solved by one of the methods explained under A.

Reduce the fractions in each of the following examples to a common denominator:—

2. $\frac{1}{2}$ and $\frac{1}{4}$.
3. $\frac{2}{3}$ and $\frac{1}{2}$.
4. $\frac{3}{4}$ and $\frac{3}{7}$.
5. $\frac{5}{6}$ and $\frac{3}{8}$.
6. $\frac{7}{9}$, $\frac{1}{6}$, and $\frac{11}{12}$.
7. $\frac{9}{10}$, $\frac{7}{15}$, and $\frac{2}{3}$.
8. $\frac{3}{7}$, $\frac{2}{5}$, and $\frac{9}{14}$.
9. $\frac{4}{9}$, $\frac{7}{8}$, and $\frac{5}{12}$.

10. $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{1}{6}$.
11. $\frac{3}{7}$, $\frac{1}{2}$, and $\frac{2}{3}$.
12. $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{7}{8}$.
13. $\frac{9}{10}$, $\frac{7}{15}$, and $\frac{1}{2}$.
14. $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{3}{4}$.
15. $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{1}{2}$.
16. $\frac{8}{9}$, $\frac{7}{12}$, $\frac{11}{18}$, and $\frac{3}{4}$.
17. $\frac{1}{4}$, $\frac{13}{20}$, $\frac{3}{8}$, and $\frac{9}{40}$.
18 $\frac{2}{9}$, $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{5}{18}$.
19. $\frac{3}{10}$, $\frac{7}{20}$, $\frac{1}{4}$, and $\frac{2}{5}$.

LESSON LVII.

A. In order that fractions may be added or subtracted, they must be simple fractions, and have a common denominator. Hence —

Complex and compound fractions must be reduced to simple fractions, and simple fractions to a common denominator, before they can be added or subtracted.

1. What is the sum of $\frac{8}{9}$ of $\frac{3}{4} + \dfrac{3\frac{1}{2}}{4\frac{1}{5}} + \frac{5}{8} + \frac{7}{12}$.

SOLUTION.—Reducing the compound and complex fractions to simple ones, we have, $\frac{8}{9}$ of $\frac{3}{4} = \frac{2}{3}$, and $\dfrac{3\frac{1}{2}}{4\frac{1}{5}} = \frac{5}{6}$. Hence, the problem becomes $\frac{2}{3} + \frac{5}{6} + \frac{5}{8} + \frac{7}{12}$. Selecting 24 as the common denominator, and reducing the fractions to twenty-fourths, as explained in Lesson LVI., gives $\frac{2}{3} + \frac{5}{6} + \frac{5}{8} + \frac{7}{12} = \frac{16}{24} + \frac{20}{24} + \frac{15}{24} + = \frac{14}{24} = 2\,\frac{17}{24}$.

2. What is the value of $\dfrac{4\frac{1}{2}}{5\frac{1}{7}}$ — $\frac{15}{22}$ of $\frac{11}{18}$?

ABBREVIATED SOLUTION.—Reducing the given fractions to simple ones and then to a common denominator, we have, $\dfrac{4\frac{1}{2}}{5\frac{1}{7}} - \frac{15}{22}$ of $\frac{11}{18} = \frac{21}{24} - \frac{10}{24} = \frac{11}{24}$.

3. Add the fractions in each of the examples under C., Lesson LVI.

4. Find the difference of the fractions in the second, third, fourth, and fifth examples under C., Lesson LVI.

5. In each example following the fifth under C., Lesson LVI., subtract the last fraction from the sum of the others.

6. $\frac{4}{5} + \frac{2}{3}$?
7. $2\frac{1}{2} + 3\frac{1}{4}$?
8. $8\frac{2}{3} + 4\frac{5}{6}$?
9. $7\frac{3}{4} + 2\frac{2}{3}$?
10. $4\frac{5}{6} + 7\frac{1}{2}$?
11. $5\frac{1}{2} + 4\frac{2}{9}$?
12. $6\frac{3}{4} - 2\frac{2}{3}$?
13. $7\frac{1}{2} - 2\frac{1}{3}$?
14. $5\frac{4}{9} + 3\frac{3}{4}$?
15. $8\frac{2}{3} - 3\frac{4}{5}$?
16. $9\frac{7}{8} - 3\frac{5}{6}$?
17. $4\frac{1}{2} - 2\frac{2}{3}$?
18. $5\frac{6}{7} + 2\frac{3}{5}$?
19. $7\frac{1}{6} - 4\frac{3}{4}$?
20. $9\frac{1}{4} + 6\frac{1}{2}$?
21. $\frac{7}{8} + \frac{3}{4} + \frac{5}{6}$?
22. $\frac{2}{3} + \frac{4}{5} + \frac{1}{2}$?
23. $\frac{7}{9} + \frac{13}{15} + \frac{2}{3}$?
24. $\frac{8}{15} + \frac{7}{10} + \frac{2}{3} + \frac{1}{6}$?
25. $\frac{1}{2}$ of $\frac{3}{4} + \frac{4}{5}$?
26. $\frac{5}{6}$ of $\frac{9}{11} + \frac{13}{22}$?
27. $8\frac{1}{4} + 8\frac{5}{9} - 6\frac{7}{12}$?
28. $4\frac{2}{5} + 6\frac{4}{9} - 7\frac{8}{15}$?
29. $\frac{5}{8}$ of $\frac{14}{15} + \frac{2}{3}$ of $\frac{1}{4}$?
30. $\dfrac{2\frac{1}{2}}{8\frac{1}{5}} + \frac{5}{6}$ of $\frac{9}{10}$?
31. $\frac{3}{5} + \dfrac{1\frac{2}{3}}{2\frac{6}{7}} + \frac{1}{2}$?
32. $\dfrac{1\frac{1}{2}}{3} + \dfrac{2\frac{2}{3}}{5\frac{1}{3}} + \dfrac{2\frac{1}{2}}{5}$?
33. $\frac{2}{7}$ of $\frac{8}{9}$ of $\frac{21}{32} + \frac{4}{9}$?
34. $8\frac{1}{2} + 3\frac{4}{5} + 7\frac{1}{3}$?
35. $\dfrac{3\frac{1}{2}}{8\frac{2}{5}} + \frac{4}{9}$ of $\frac{3}{8}$?

B. 1. Rufus bought a slate for $\frac{1}{4}$ of a dollar, a writing book for $\frac{1}{8}$ of a dollar, a geography for $\frac{1}{2}$ of a dollar, and an atlas for $\frac{7}{16}$ of a dollar. What was the cost of the whole?

2. Edward spends $1\frac{1}{3}$ hours each day in studying history, $1\frac{3}{4}$ hours in studying geography, and $1\frac{5}{6}$ hours in studying grammar. How many hours does he spend in studying all these branches?

3. A man bought a large pine-apple and gave $\frac{1}{4}$ of it to Sarah, $\frac{1}{5}$ of it to Jane, $\frac{1}{10}$ of it to Susan, $\frac{3}{20}$ of it to Maria, and the rest to Emma. What part of it did he give to Emma?

4. I spent $\frac{2}{7}$ of my money for land, $\frac{5}{14}$ of it for buildings, and put the rest at interest. What part of it did I put at interest?

5. I bought an umbrella for $\$1\frac{3}{8}$, and a pair of shoes for $\$2\frac{4}{5}$. How much did both cost? How much more did the shoes cost than the umbrella?

6. A farmer sold $6\frac{3}{4}$ tons of hay, and then had $8\frac{5}{8}$ tons left. How many tons had he at first?

7. A man walked from Dedham to Boston, a distance of 10 miles, in 3 hours. He walked $2\frac{4}{5}$ miles the first hour, and $3\frac{7}{10}$ miles the second. How far did he walk in the third hour?

8. Mr. Wheelock bought a book for $\$1\frac{5}{8}$, and a ream of paper for $\$2\frac{4}{5}$, giving in payment a five-dollar bill. How much money ought he to receive back?

9. Mr. Nichols's orchard contains $3\frac{2}{3}$ acres, and his house-lot contains $\frac{3}{4}$ of an acre. How many acres do both contain, and how many more acres are there in his orchard than in his house-lot?

10. Mr. Turner bought 4 loads of hay. The first weighed $\frac{3}{4}$ of a ton, the second weighed $\frac{1}{2}$ of a ton, the third weighed $\frac{5}{8}$ of a ton, and the fourth weighed $\frac{3}{5}$ of a ton. What did they all weigh?

11. I bought $\frac{3}{4}$ of a yard of silk velvet at \$7 per yard, and $\frac{5}{8}$ of a yard of satin at \$6 per yard. What did both cost?

12. I sold 7 barrels of apples at $\$2\frac{1}{8}$ per barrel, receiving in payment 4 yards of cloth at $\$3\frac{2}{3}$ per yard, and the rest in money. How much money did I receive?

13. This morning I had $\$10\frac{1}{4}$, but I have since paid away $\$6\frac{7}{12}$. How much have I left?

14. A farmer gathered $7\frac{1}{2}$ barrels of russets, $8\frac{2}{3}$ barrels of pippins, $5\frac{5}{6}$ barrels of greenings, and $9\frac{3}{4}$ barrels of sweetings. How many barrels did he gather in all?

15. Arthur, Richard, and Edwin were talking about their money. Arthur said that he had $\$4\frac{1}{3}$. "Then," said Richard, "I have $\$2\frac{5}{12}$ more than you have." Edwin thought a moment,

then said, "If I had $8½ more than I now have, I should have as much as both of you together. How many did Richard have? How many did Edwin have?

16. Rufus spent ½ of his money for writing paper, ⅓ of it for pens, and the rest, which was 4 cents, for a pencil. What part of his money did he spend for a pencil? How many cents did he spend in all, and how many for each article?

17. A farmer has ½ of his sheep in one pasture, ⅖ of them in another, and the rest, 6 sheep, in another. How many has he in all, and how many in each pasture?

18. Benjamin being asked his age, replied, "I have spent 5/18 of my life in Brooklyn, ⅙ of it in New York, 2/9 of it in Baltimore, and the rest, 6 years, in Cincinnati. What was his age?

19. A drover says that if he sells ⅓ of his sheep to one man, and ¼ of them to another, he shall sell 6 more to the first man than to the second. How many sheep has he?

20. James and George were talking about their ages. James said that ½ of his age exceeded ⅓ of it by 1½ years; to which George replied, "Then you are only ⅗ as old as I am." What was the age of each boy?

21. 3½ times a certain number exceeds 2 times the number by 12. What is the number?

22. If I could sell my cow for 13 dollars more than 3 times what she cost me, I should receive $100 for her. How much did she cost me?

23. There is an orchard in which 2/9 of the trees bear peaches, ⅙ bear cherries, ⅓ bear apples, and the rest bear pears. Now, if there are 7 more apple trees than peach trees, how many trees are there in the orchard, and how many of each kind?

24. Mr. Jones and Mr. French traded in company. Mr. Jones put in $8 as often as Mr. French put in $4. When they came

to divide the gain, it was found that Mr. French's share was $8 more than Mr. Jones's. How much did they gain, and what was the share of each?

LESSON LVIII.

A. 1. Of what denominations is the number 427 composed?

Ans.—The number 427 is composed of 4 hundreds, 2 tens, and 7 units.

In the same way tell of what denominations each of the following numbers is composed:—

2.	678.	5.	5276.	8.	2008.
3.	982.	6.	3028.	9.	3254.
4.	201.	7.	1406.	10.	6897.

11. Explain the use of the figures of the above numbers, as in the following—

Model.—In 427, the 7 marks the units' place, and shows that there are 7 units; the 2 marks the tens' place, and shows that there are 2 tens; the 4 marks the hundreds' place, and shows that there are 4 hundreds.

12. Give the value of each figure of the above numbers, as in the following—

Model.—In 427, the 7 = 7 units; the 2 = 2 tens, or 20 units; the 4 = 4 hundreds, or 40 tens, or 400 units.

B. The foregoing illustrations show that—

The value of each figure is ten times the value it would have if it stood one place farther to the right, and one-tenth of the value it would have if it stood one place farther to the left.

1. Compare the values expressed by the 2's of 222.

Ans.—The first, or right hand 2, expresses $\frac{1}{10}$ the value of the second 2, and $\frac{1}{100}$ the value of the third 2; the second, or middle 2, expresses 10 times the value of the second 2, and $\frac{1}{10}$ the value of the third; the third, or left-hand 2, expresses 10 times the value of the second 2, and 100 times the value of the first.

Compare in the same way the figures of each of the following numbers:—

2. 333.
3. 111.
4. 444.
5. 5555.
6. 9999.
7. 6006.
8. 808.
9. 7777.
10. 2202.

C. Marking the places by a period, or decimal point (see Lesson XXII., C.), we may make new places at the right of the point, by calling the first tenths, the second hundredths, the third thousandths, &c.

Thus: 42.37 = 4 tens, 2 units, 3 tenths, and 7 hundredths; .348 = 3 tenths, 4 hundredths, and 8 thousandths.

Name the denominations of the figures of the following numbers:—

1. 23.47
2. 6.325
3. .3698
4. 1.46
5. .008
6. .06
7. 4.596
8. 1.037
9. .027
10. 2.7
11. 4.06
12. 30.03

D. Such numbers are read by first reading the figures at the left of the point, *as though they stood alone*,—and then reading the figures at the right of the point, *as though they stood alone, naming afterwards the denomination of the right-hand figure.*

ILLUSTRATIONS.—42.37 = 42 and 37 one-hundredths = $42\frac{37}{100}$. .348 = 348 thousandths = $\frac{348}{1000}$, &c.

In the same way, read each of the numbers under C.

Numbers expressed by figures written both at the right and the left of the point, were, in the above form, read as *mixed numbers.* They may, with equal propriety, be read as *improper fractions.*

Illustrations.—$42.37 = 42\frac{37}{100} = \frac{4237}{100}$; $5.6 = 5\frac{6}{10} = \frac{56}{10}$, &c.

Read each number under C., which is greater than 1, as an improper fraction.

E. 1. If the decimal point of any number be removed one place farther towards the right, or, which is the same thing, the figures be removed one place towards the left, each figure will represent 10 times as large a value as before; while if the point be removed one place farther towards the left, or, which is the same thing, the figures be removed one place toward the right, each figure will represent one-tenth as large a value as before.

2. A similar change of two places, would multiply or divide a number by 100,—of three places, by 1000, &c., &c.

3. Hence—*To multiply a number by* 10, *it is only necessary to remove the point one place farther towards the right; to multiply by* 100, *remove it two places, &c., &c.*

4. So, *to divide a number by* 10, *remove the point one place towards the left; to divide by* 100, *remove it two places, &c., &c.*

5. *If there are not figures enough at the right or left of the point to make these changes, annex or prefix zeroes to make up the deficiency.*

Illustrations.

$46 \times 10 = 460$	$46 \div 10 = 4.6$
$3.7 \times. 10 = 37$	$4 \div 10 = .4$
$5.86 \times 10 = 58.6$	$067 \div 10 = .0067$
$234 \times 100 = 23400$	$634 \div 100 = 5.34$
$57.8 \times 100 = 5780$	$.85 \div 100 = .0085$
$6.294 \times 100 = 629.4$	$6.9 \div 100 = .069$

Multiply each of the following numbers by 10 :—

1. 84
2. 5.6
3. .63
4. 5.24
5. 63.7
6. 286
7. 2847
8. 54.09
9. 3.275
10. 8.246
11. .9374
12. 23.16

13. Multiply each of the above numbers by 100, and then by 1000.

14. Divide each of the above numbers by 10, then by 100, then by 1000.

15. Find $\frac{1}{10}$ of each of the above numbers, then $\frac{1}{100}$, then $\frac{1}{1000}$.

F. 1. What is .03 of 145.6 ?

Solution.—Since .03 of a number equals 3 times $\frac{1}{100}$ of that number, it may be found by removing the point two places further towards the left and multiplying by 3, which would give .03 of 145.6 = 3 times 1.456 = 4.368.

Note.—Probably it will be better to have the pupil perform most of these questions on his slate.

2. .4 of 6.8 ?
3. .05 of 27 ?
4. .3 of 56 ?
5. 1.3 of 6.7 ?
6. .07 of 5.6 ?
7. .02 of 176 ?
8. .2 of .06 ?
9. .25 of 183 ?
10. .003 of 279 ?
11. .004 of 8.27 ?
12. 1.2 of 43 ?
13. 1.42 of .687 ?

G. 1. What is the quotient of 4.8 ÷ .006 ?

Since the quotient of a number divided by .006 equals (Lesson LV.) $\frac{1}{6}$ of 1000 times that number, it may be found by removing the point three places toward the right and dividing by 6, which would give 4.8 ÷ .006 = $\frac{1}{6}$ of 4800 = 800.

2. 4.25 ÷ 5 ?
3. 25 ÷ .05 ?
4. .06 ÷ .006 ?
5. .0144 ÷ .12 ?
6. 32 ÷ .008 ?
7. 2.76 ÷ 1.2 ?
8. 2.76 ÷ .12 ?
9. 2.76 ÷ .012 ?
10. .325 ÷ 25 ?
11. 36 ÷ .006 ?
12. 49 ÷ 4.9 ?
13. 37 ÷ .037 ?

H. The term PER CENT is often used in place of *one hundredths.*

Thus, 6 per cent = .06, or $\frac{6}{100}$; 9 per cent = .09, or $\frac{9}{100}$, &c., &c.

1. I gathered 48 bushels of apples, receiving 12 per cent of them for my labor. How many bushels did I receive?

2. I bought a sleigh for $16.20, and paid a sum equal to 8 per cent of the cost for having it repaired. How much did I pay for having it repaired?

3. A man who had 87 bushels of apples, sold .7 of them and kept the rest. How many bushels did he sell? How many did he keep?

4. A father left, at his death, 97 acres of land, to be so divided that his widow should have .4 of it, his oldest son .3, his youngest son .2, and his daughter the rest. What was the share of each?

5. George received 9 per cent of $144, and William received 6 per cent of $216. Which received the most?

6. A trader bought a lot of goods for $36, and sold them so as to gain 10 per cent of the cost. What was his gain, and for how much did he sell them?

7. Bought goods for $300, and sold them so as to gain 15 per cent. What was my gain?

8. I gave $28.60 for a lot of goods, but I was obliged to sell them so as to lose 8 per cent. How many dollars did I lose, and for how many dollars did I sell them?

9. Mr. Brown bought a horse for $150, and sold him at an advance, or gain, of 12 per cent. What was his gain?

10. I bought a carriage for $175, and, after paying 12 per cent of the cost for repairing it, I sold it for $225. Did I gain or lose, and how much?

J. The money which men charge for their services in buying or selling goods for others, is called COMMISSION, and is usually a certain per cent of the cost of the goods bought, and of the money received for those sold.

1. Mr. Clarke sold a lot of goods for Mr. Davis for $500, at a commission of 3 per cent. What did his commission amount to, and how much money would be left for Mr. Davis?

2. I sold a lot of goods for $250, at a commission of 4 per cent. What did my commission amount to, and what would be left for the owner of the goods?

3. A commission merchant sold 35 barrels of flour, at $8 per barrel, receiving a commission of 2 per cent. What was his commission?

4. I bought $360 worth of cloth for Mr. Arnold, charging him a commission of 2 per cent. What was my commission, and what ought Mr. Arnold to pay me for the cloth and my commission?

5. George bought a jack-knife for James for 75 cents, charging a commission of 8 per cent. How much ought James to pay for the knife and George's commission?

6. Mr. Greene bought a lot of shoes for Mr. Gardner, for which he paid $120, and charged 3½ per cent commission. What ought Mr. Gardner to pay for the shoes and Mr. Greene's commission?

7. By selling a horse for 20 per cent more than he cost, I gained $30. What did he cost, and for how much did I sell him?

SUGGESTION.— The given per cent can often be reduced to lower terms. Thus, 20 per cent. $= \frac{20}{100} = \frac{1}{5}$; 16⅔ per cent. $= \frac{16\frac{2}{3}}{100} = \frac{1}{6}$, &c.

8. By selling a lot of merchandise at an advance of 12½ per cent, I gained $9.50. What did it cost me, and for how much did I sell it?

9. My commission of $3 per cent for selling a lot of goods was *$15. For how* much did I sell them?

10. I lost 25 per cent of the cost of a horse by selling him for $120. What per cent of his cost did I receive? How many dollars did he cost me? How many dollars did I lose?

11. I gained 16⅔ per cent of the cost of a horse by selling him for $140. What was his cost, and how many dollars did I gain?

12. By selling some cloth at 24 cents per yard, I should gain 5 per cent more than I should by selling it at 28 cents per yard. What was its cost?

13. By selling cloth at 12½ cents per yard, I gain 25 per cent. For how much should I sell it to gain 50 per cent?

LESSON LIX.

A. 1. If I should have the use of another man's horse for a day, or a week, I ought to pay for it; or if I should occupy a house or a store belonging to another, I ought to pay rent for the use of it. In like manner, if I should borrow a sum of money, I ought to pay for the use of it.

2. Money thus paid for the use of money, is called **Interest**.

3. The money lent or used is called the **Principal**, and the principal and interest together, form the **Amount**.

Illustrations.—If I should pay $3 for the privilege of using $100 for six months, the $3 would be the *interest* of the $100 for 6 months; the $100 would be the *principal*, and $100 + $3, or $103, would be the *amount*.

4. The interest is usually a certain number of one hundredths of the principal for each year it is used. This number of one hundredths is called the **Rate per cent**, or simply the **Rate**.

Illustration.—If a man is to pay a sum equal to $\frac{6}{100}$ of the principal for each year he uses it, the rate is 6 per cent.

5. In computing interest, a month is reckoned at 30 days.

B. 1. What is the interest of $8 for 2 years 9 mo., at 4 per cent.?

Solution.—At 4 per cent. per year, the interest for 2 yr. 9 mo., or 2¾ years, must be 2¾ times 4 per cent., or 11 per cent. of the principal. 11 per cent. of $8 = 11 times 8 cents = 88 cents, or $.88 = the answer.

What is the interest —

2. Of $7 for 2 yr., at 6 per cent?
3. Of $9 for 3 yr., at 5 per cent?
4. Of $18 for 6 mo., at 6 per cent?
5. Of $243 for 4 mo., at 6 per cent?
6. Of $43.21 for 1 yr. 10 mo. at 6 per cent?
7. Of $52.30 for 2 yr. 6 mo., at 4 per cent?
8. Of $132 for 1 yr., at 7 per cent?
9. Of $937 for 8 mo., at 6 per cent?
10. Of $42.73 for 2 yr., at 4½ per cent?
11. Of $23.17 for 1 mo., at 6 per cent?
12. Of $24.36 for 9 mo., at 8 per cent?
13. Of $53.27 for 1 yr. 4 mo., at 6 per cent?

C. Interest is more frequently reckoned at 6 per cent per year, than at any other rate. Hence, in all the following examples and explanations, interest should be reckoned at 6 per cent, unless otherwise stated.

2 months being ⅙ of a year, interest for 2 months at 6 per cent. must equal ⅙ of 6 per cent, or 1 per cent of the principal, which may be found by removing the decimal point 2 places to the left, and is as many cents as there are dollars in the principal.

At 6 per cent per year, what is the interest for 2 months of —

1. $37?
2. $58?
3. $49?
4. $657?
5. $938?
6. $8238?
7. $35.75?
8. $123.79?
9. $437.28?

10. What is the amount of each of the above?

D. Interest for 2 months being 1 per cent of the principal, interest for 100 times 2 months, or 200 months, or 16 years 8 months, must be 100 per cent of the principal, which is the principal itself.

From this we compute the following table:—

At 6 per cent per year, interest for —

200 mo., or 16 yr. 8 mo. = principal.

$\frac{1}{2}$ of 200 mo., or 8 yr. 4 mo. = $\frac{1}{2}$ of prin.

$\frac{1}{3}$ of 200 mo., or $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da. = $\frac{1}{3}$ of prin.

$\frac{1}{4}$ of 200 mo., or 50 mo., or 4 yr. 2 mo. = $\frac{1}{4}$ of prin.

$\frac{1}{5}$ of 200 mo., or 40 mo., or 3 yr. 4 mo. = $\frac{1}{5}$ of prin.

$\frac{1}{6}$ of 200 mo., or $33\frac{1}{3}$ mo., or 2 yr. 9 mo. 10 da. = $\frac{1}{6}$ of prin.

$\frac{1}{8}$ of 200 mo., or 25 mo., or 2 yr. 1 mo. = $\frac{1}{8}$ of prin.

$\frac{1}{10}$ of 200 mo., or 20 mo., or 1 yr. 8 mo. = $\frac{1}{10}$ of prin.

$\frac{1}{12}$ of 200 mo., or $16\frac{2}{3}$ mo., or 1 yr. 4 mo. 20 da. = $\frac{1}{12}$ of prin.

$\frac{1}{15}$ of 200 mo., or $13\frac{1}{3}$ mo., or 1 yr., 1 mo. 10 da. = $\frac{1}{15}$ of prin.

$\frac{1}{16}$ of 200 mo., or $12\frac{1}{2}$ mo., or 1 yr. 15 da. = $\frac{1}{16}$ of prin.

1. What is the interest of $60 for each time mentioned in the table?

Ans.—The interest of $60 for 200 mo., or 16 yr. 8 mo. = $60; for 100 mo., or 8 yr. 4 mo. = $\frac{1}{2}$ of $60 = $30; for $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da. = $\frac{1}{3}$ of $60 = $20, &c., &c.

2. What is the interest of $36 for each time mentioned in the table? 3. Of $48.72?

What is the interest of —

4. $40 for 100 mo.? 6. $64 for 4 yr. 2 mo.?

5. $48 for $12\frac{1}{2}$ mo.? 7. $24.60 for 5 yr. 6 mo. 20 da.?

8. \$66 for $16\frac{2}{3}$ mo.?
9. \$24.36 for $66\frac{2}{3}$ mo.?
10. \$16.98 for 25 mo.?
11. \$84.60 for 20 mo.?
12. \$42 for 50 mo.?
13. \$37 for 40 mo.?
14. \$54.72 for $33\frac{1}{3}$ mo.?
15. \$75.15 for $13\frac{1}{3}$ mo.?
16. \$16.64 for 1 yr. 15 da.?
17. \$25.75 for 3 yr. 4 mo.?
18. \$44.36 for 1 yr. 8 mo.?
19. \$16.24 for 8 yr. 4 mo.?
20. \$44 for 2 yr. 1 mo.?
21. \$60.45 for 1 yr. 1 mo. 10 da.?
22. \$43.78 for 16 yr. 8 mo.?
23. \$75 for 2 yr. 9 mo. 10 da.?

24. What is the amount of each of the above?

E. The interest for 20 mo., or 1 yr. 8mo., being $\frac{1}{10}$ of the principal, may be found by removing the decimal point 1 place to the left, and is as many dimes as there are dollars in the principal.

Hence the interest for —

$\frac{1}{2}$ of 20 mo., or 10 mo., = $\frac{1}{2}$ of $\frac{1}{10}$ of principal.
$\frac{1}{3}$ of 20 mo., or $6\frac{2}{3}$ mo., or 6 mo. 20 da. = $\frac{1}{3}$ of $\frac{1}{10}$ of prin.
$\frac{1}{4}$ of 20 mo., or 5 mo. = $\frac{1}{4}$ of $\frac{1}{10}$ of prin.
$\frac{1}{5}$ of 20 mo., or 4 mo. = $\frac{1}{5}$ of $\frac{1}{10}$ of prin.
$\frac{1}{6}$ of 20 mo., or $3\frac{1}{3}$ mo., or 3 mo. 10 da. = $\frac{1}{6}$ of $\frac{1}{10}$ of prin.
$\frac{1}{8}$ of 20 mo., or $2\frac{1}{2}$ mo., or 2 mo. 15 da. = $\frac{1}{8}$ of $\frac{1}{10}$ of prin.
$\frac{1}{12}$ of 20 mo., or $1\frac{2}{3}$ mo., or 1 mo. 20 da. = $\frac{1}{12}$ of $\frac{1}{10}$ of prin.
$\frac{1}{15}$ of 20 mo., or $1\frac{1}{3}$ mo., or 1 mo. 10 da. = $\frac{1}{15}$ of $\frac{1}{10}$ of prin.

1. What is the interest of \$24 for each time mentioned in the table?

Ans.—The interest of \$24 for 10 mo. = $\frac{1}{2}$ of \$2.40 = \$1.20; for $6\frac{2}{3}$ mo., or 6 mo. 20 da. = $\frac{1}{3}$ of \$2.40 = \$.80, &c.

2. What is the interest of \$120 for each time

mentioned in the table? 3. Of $7.50? 4. Of $4.86?

What is the interest of—

5. $72 for 2½ mo, ?
6. $60 for 1⅓ mo.?
7. $486 for 1⅔ mo.?
8. $15 for 10 mo.?
9. $2.40 for 6⅔ mo.?
10. $64.50 for 4 mo.?
11. $36.60 for 3⅓ mo.?
12. $483.60 for 1 mo. 20 da.?
13. $27 for 5 mo.?
14. $7.50 for 2 mo. 15 da.?
15. $74.10 for 3 mo. 10 da.?
16. $55 for 1 mo. 10 da.?
17. $1.86 for 6 mo. 20 da.?
18. $54.20 for 4 mo.?

19. What is the amount of each of the above?

F. The interest for 2 months, or 60 days, being $\frac{1}{100}$ of the principal, it follows that the interest for—

$\frac{1}{2}$ of 2 mo., or 1 mo., or 30 da. = $\frac{1}{2}$ of $\frac{1}{100}$ of the principal.
$\frac{1}{3}$ of 2 mo., or 20 da. = $\frac{1}{3}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{4}$ of 2 mo., or 15 da. = $\frac{1}{4}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{5}$ of 2 mo., or 12 da. = $\frac{1}{5}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{6}$ of 2 mo., or 10 da. = $\frac{1}{6}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{10}$ of 2 mo., or 6 da. = $\frac{1}{10}$ of $\frac{1}{100}$ or $\frac{1}{1000}$ of the prin.
$\frac{1}{12}$ of 2 mo., or 5 da. = $\frac{1}{12}$ of $\frac{1}{100}$ of the prin.
$\frac{1}{2}$ of 6 da., or 3 da. = $\frac{1}{2}$ of $\frac{1}{1000}$ of the prin.
$\frac{1}{3}$ of 6 da., or 2 da. = $\frac{1}{3}$ of $\frac{1}{1000}$ of the prin.
$\frac{1}{6}$ of 6 da., or 1 da. = $\frac{1}{6}$ of $\frac{1}{1000}$ of the prin.

1. What is the interest of $432 for each time mentioned in table?

SOLUTION.—The interest of $432 for 2 mo. is $4.32; for 1 mo. is ½ of $4.32, which is $2.16, &c., &c., * * * for 6 days, is $.432; for 3 days is ½ of $.432, which is $.216, &c., &c.

2. What is the interest of $360 for each time mentioned in the table? 3. Of $60.30?

What is the interest of—

4. $42 for 20 da.?
5. $36.24 for 15 da.?
6. $48 for 10 da.?
7. $89 for 6 da.?
8. $174 for 3 da.?
9. $192 for 5 da.?
10. $43.50 for 12 da.?
11. $86.37 for 30 da.
12. $228 for 1 da.?
13. $234 for 2 da.?

14. What is the amount of each of the above?

G. The foregoing principles furnish short and expeditious methods of computing interest for any time whatever.

1. What is the interest of $72.60 for 8 mo. 20 da.?

1st Solution.—8 mo. 20 da. = 6 mo. 20 da. + 2 mo. The interest of $72.60 for 6 mo. 20 da. = ⅓ of $7.26 = $2.42, and the interest for 2 mo. = $.726, which, added to $2.42 = $3.146 = Ans.

2d Solution.—8 mo. 20 da. = 10 mo. — 1 mo. 10 da. The interest $72.60 for 10 mo. = ½ of $7.26 = $3.63, and the interest for 1 mo. 10 da. = $\frac{1}{15}$ of $7.26 = $.484, which, subtracted from $3.63 = $3.146 = Ans.

3d Solution. — 8 mo. 20 da. = 8 mo. + 20 da. The interest of $72.60 for 8 mo. or 4 times 2 mo. = 4 per cent of $72.60 = $2.904, and the interest for 20 da. = ⅓ of $.726 = $.242, which, added to $2.904 = $3.146 = Ans.

The work can be written as follows:—

1st Solution.

$72.60 = prin.
2.42 = int. 6 mo. 20 da.
.726 = int. 2 mo.
$3.146 = int. 8 mo. 20 da.

2d Solution.

$72.60 = prin.
3.63 = int. 10 mo.
.484 = int. 1 mo. 10 da.
$3.146 = int. 8 mo. 20 da.

The form for the third solution would be similar to these.

What is the interest of —

2. $90 for 3 mo. 16 da.?
3. $128 for 22 mo. 15 da.?
4. $64 for 2 mo. 10 da.?
5. $32 for 5 mo. 15 da.?
6. $120.90 for 3 mo. 20 da.?
7. $88.24 for 5 mo. 6 da.?
8. $72.96 for 1 mo. 26 da.?
9. $500 for 9 mo. 24 da.?
10. $1000 for 8 mo. 29 da.?
11. $54.24 for 7 mo. 15 da.?
12. $150 for 35 mo. 10 da.?
13. $184 for 2 yr. 3 mo.?
14. $96 for 52 mo. 15 da.?
15. $186.60 for 7 mo.?
16. $28.16 for 2 yr. 6 mo.?
17. $384 for 19 mo. 27 da.?
18. $30.24 for 6 mo. 17 da.?
19. $450.36 for 3 mo. 15 da.?

H. Business men often use such methods as the following in connexion with those already explained:—

At 6 per cent. per year the interest of $2 for 1 month is 1 cent. Hence —

The interest of $2 *is* 1 *cent per month.*

The interest of $20 *is* 1 *dime per month.*

The interest of $200 *is* 1 *dollar per month.*

1. What is the interest of $2 for each of the following times?

3 mo.?
9 mo.?
15 mo.?
2 yr. 3 mo.?
1 yr. 5 mo.?
2 yr. 1½ mo.?
15⅓ mo.?
4 mo. 10 da.?
2 yr. 7 mo.?

What is the interest for each of the above times of —

2. $1?
3. $6?
4. $8?
5. $20?
6. $5?
7. $10?
8. $200?
9. $50?
10. $500?
11. $14?
12. $80?
13. $800?

At 6 per cent. per year —

The interest of $6 *is* 1 *mill per day.*

The interest of $60 *is* 1 *cent per day.*

The interest of $600 *is* 1 *dime per day.*

The interest of $6000 *is* 1 *dollar per day.*

What is the interest of $66 for each of the following times?

3 da.?
7 da.?
19 da.?
1 mo. 3 da., or 33 da.?
1 mo. 17 da.?
2 mo. 25 da.?
3 mo. 6 da.?
5 mo. 12 da.?
4 mo. 9 da.?

What is the interest for each of the above times of —

1. $60?
2. $30?
3. $20?
4. $120?
5. $600?
6. $300?
7. $150?
8. $1800?
9. $6000?
10. $1000?
11. $1500?
12. $500?

LESSON LX.

MISCELLANEOUS PROBLEMS.

1. 7 times 8, plus 4, divided by 5, multiplied by 3, minus 4, minus 8, divided by 2, divided by 3, multiplied by 9, multiplied by 2 = how many times?

2. Multiply $\frac{1}{2}$ of 18 by $\frac{3}{4}$ of 8, add $\frac{1}{3}$ of 27, divide by $\frac{1}{4}$ of 28, add $\frac{1}{11}$ of 33, and square the number.

3. If $\frac{6}{7}$ of a yard of cloth worth 14 cents per yard, are given for $\frac{3}{5}$ of a pound of chocolate, how many pounds of coffee at 12 *cents per pound* should be given for $3\frac{1}{2}$ pounds of chocolate?

4. If I should expend the sum of $9 + $8 + $5 + $9 + $4 + $7 for flour at $7 per barrel, and sell the flour at $8 per barrel, then expend the proceeds for cloth at $3 per yard, and sell the cloth for $4 per yard, and then, after spending $10, and losing $6, should expend the remainder for tea at the rate of 3 pounds for $2, how many pounds of tea should I buy?

5. Find the cost of 1¾ lb. coffee at 16 cents per lb., 2⅓ lb. raisins at 8 cents per lb., 2⅔ lb. figs at 15 cents per lb., 4⅓ lb. sugar at 9 cents per lb., 1⅗ lb. tea at 40 cents per lb., and ¾ lb. cotton at 32 cents per lb. Make out a bill on the supposition that you sold the above articles to one of your school-mates.

6. What must be the length of the side of a square field containing ½ as many square rods as a field 9 rods long and 8 rods wide?

7. Arthur sold a certain number of apples at the rate of 2 for a cent, and Robert sold as many at the rate of 3 for a cent. Arthur received 12 cents more than Robert. How many apples did each sell?

8. A thief drew ¼ of the wine out of a certain cask, and, to escape detection, filled it with water. The next night he drew out ¼ of the contents of the cask, and again filled it with water. How many gills of wine will there now be in each gallon of the mixture?

9. 2½ times a certain number added to ⅓ of that number is 5½ less than 3 times the number. What is the number?

10. A lady being asked her age, replied, "My father is 30 years older than my sister Sarah, and 8⅓ times the difference between their ages is 5 times my father's age. Now, if you will tell how old my father and sister are, I will tell you how to find my age?" A correct answer having been given, the lady said, "To 3 times my father's age, add 6 times my sister's age, and

you will obtain a sum ⅓ of which is 9 years more than 4½ times my age?" What was the age of each?

11. A teacher wishing to obtain a black-board 15 ft. long and 6 ft. wide, bought boards for the purpose at 2½ cents per square foot. He hired a carpenter to make it, paying him 75 cents for his work. He paid 11 cents per square yard to have it painted and varnished, and it cost 25 cents to have it brought to the school-room and put up. What was the whole cost?

12. My parlor and sitting-room are each 5 yards wide, but my parlor is 2 yards longer than my sitting-room. The floor of my sitting-room contains 30 square yards. What is the length of my parlor floor, and how many square yards does it contain?

13. David said to Harry, "If ½ the sum of our ages be added to ½ of your age, the same will equal ¾ of my age, and I am 12 years older than you are. What was the age of each of the boys?

14. Mr. Warren bought a cask of oil at $1.20 per gallon, but ⅕ of it leaked out. For how much per gallon must he sell the rest so as neither to gain nor lose?

15. Mr. Allen owes Mr. Mason 62 cents, and the only coins he has are 1 half-dollar, 1 quarter-dollar, 1 half-dime, and 2 three-cent pieces, while the only coins Mr. Mason has are 4 half-dollars, 5 dimes, and 2 cents. How can change be made so that the debt may be paid with these coins?

16. What number added to ⅔ of itself equals 36 more than ½ of the number?

17. A man sold 6 barrels of apples and 2 barrels of pears for $23, receiving twice as much per barrel for the pears as for the apples. How many dollars did he receive for each?

18. By selling cloth at $3.50 per yard, I lose 12½ per cent of its cost. How many dollars should I lose on each yard by selling it at $3 per yard?

19. I sold ½ of a lot of grain for what ⅔ of it cost, thereby gaining $16. How much did the entire lot cost me?

20. A. and B. traded in company. A. put in $360, and B. put in ⅔ of ⅐ of $\frac{5}{9}$ of 42 times ½ as much as A. They gained a a sum equal to ⅔ of their joint stock. How much did they gain, and what was the share of each?

21. If Mr. Walton's blackboard were 2 ft. wider than it now is, it would contain 26 more square feet, but if it were 2 feet longer, it would contain 11 more square feet. How many square feet does it contain?

22. George has money enough to buy 2½ quarts of chestnuts, Rufus has twice as much as George, and Edward has ⅓ as much as Rufus. They all have 57 cents. How much are the chestnuts worth per quart, and how many cents has each of the boys?

23. The interest of Mr. Butler's money for 5 yr. 6 mo. 20 da., at 6 per cent, will equal $8000. How much money has he?

24. If a pound of rice is worth ⅔ as much as a pound of sugar, and 6 lb. of rice and 10 lb. of sugar are worth $1.26, how much are 5 lb. of rice and 7 lb. of sugar worth?

25. Why is it that if we multiply any number whatever by 3, add 7 to the product, add the first number taken to this, add 9 to this, divide this by 4, add 3 to this, and then subtract from this the first number taken, the result will always be 7?

26. By selling cloth at $1.25 per yard, I lose 16⅔ per cent. For how much per yard must I sell it to gain 20 per cent?

27. There are ⅔ as many acres in my orchard as there are in my pasture, and ¼ as many in my garden as in my orchard. If there are 17 acres in all, how many are there in each lot?

28. I bought a lot of goods for $600, and after keeping them 1 month 17 days, I sold them for $650. Now, allowing that I had to pay interest on the money invested, at the rate of 6 per cent, what was my net gain?

29. A man bought a cask of wine, but ⅔ of it leaked out. put in as much water as there was wine remaining, and sold mixture at the same price per gallon that he gave for it. W part of the cost did he lose?

30. After paying $3 more than ½ of my money to one man, $6 more than ½ of what I had left to another, I had $7 left. much did I have at first?

31. I sold 10 bushels of corn for Mr. Austin, and 8 bu for Mr. Brown, receiving $11 for the lot. Now, allowing Mr. Austin's corn is worth 20 per cent more per bushel than Brown's, and that I am to receive $1 for my services, how money ought I to pay to each?

THE END.

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